

Quality Management in the Bosch Group | Technical Statistics

2. Basic Concepts of Technical Statistics

Discrete Characteristics



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Quality Management in the Bosch Group
Technical Statistics

Booklet 2 – Basic Principles of Technical Statistics:
Discrete Characteristics

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Preface

Before the computer was invented and comfortable statistical programs were developed, the graphical display and evaluations in the present work had to be done by hand using special forms and statistical tables.

In the interest of deep understanding, it is still sensible today to reproduce the respective procedures in detail, for example when studying statistics or completing training. For this reason, the procedures are described in such a way that allows them to be evaluated by hand without the use of a calculator.



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1 Introduction

This book provides insight into the statistics of discrete characteristics. The word discrete (Latin *discretus*: separate) in this context means: values or components that are distinguishable, definable, separate from one another, taken from a countable set of elements. In order to understand what it means within the framework of statistics, it is necessary to first consider some terms that are defined in DIN 55350-12. The definitions quoted from this standard are noted with quotation marks as follows.

Characteristic

“Feature for the recognition and distinction of units.”

Quantitative characteristic

“Characteristic whose values are allocated to a scale with defined distances.”

All physical quantities are quantitative characteristics, for example length, volume, angle, weight, mass, temperature, tension, current, time, speed.

Continuous Characteristic

“Quantitative characteristic whose co-domain (range of values) is uncountably infinite.”

The value of such a characteristic is always given as a product of numerical value and unit.

EXAMPLES:

- *Length: 12.54561... m,*
- *Diameter: 3.532... mm.*

The numerical value of the characteristic "length", for example, can take on every value between 12 and 13; mathematically expressed: every real number on the interval between 12 and 13.

In reality, no value of such a continuous characteristic can be measured with infinite accuracy. The number of decimal places is always limited by the number of available digits that can be displayed. In addition, properties of the measurement process can limit the number of useful decimal places.

Discrete characteristic

“Quantitative characteristic whose co-domain is finite or countably infinite.”

EXAMPLES:

- *Mass in kg (without decimal places).*
- *Body weight in cm (without decimal places).*
- *Number of "6" when rolling a die 100 times. The result can take on all of the values from 0 to 100. The number is finite.*
- *Number of lightning strikes in Germany during a given year (one can also locate and, of course, record atmospheric lightning using antennae). This number can take on a countably infinite number of values (0, 1, 2, 3, ...), although very large values will appear with decreasing probability.*



Countable characteristic

“Special discrete characteristic whose co-domain is given by the set of natural numbers including zero (0, 1, 2, ...) or a subset of this set.”

All previously mentioned examples for discrete characteristics are countable characteristics.

Qualitative characteristic

“Characteristic whose values are allocated to a scale without defined distances.”

Nominal characteristic

“Qualitative characteristic whose values have no quantitative relation to each other.”

EXAMPLE: The characteristic "color" with the values "red", "yellow", "green".

The values of these characteristics can only be classified and differentiated (is equal, is not equal).

“The value of a nominal characteristic is often also called attribute. A nominal characteristic with only two, mutually exclusive values is called a dichotomous characteristic, a binary characteristic or an alternative characteristic. It can only take on one of two mutually exclusive values.”

EXAMPLES:

- *good / bad,*
- *inside / outside the tolerance range (OK / not OK),*
- *defective / not defective,*
- *present / not present.*

Such data is, for instance, gained using limit gauges or by visual assessment with boundary samples. It is, however, also possible to define a “discrete” continuous characteristic by first measuring it and then classifying the measured value using a category “inside/outside the tolerance range”. However, this is rarely sensible due to the associated loss of information. If measurement values exist for a continuous characteristic, they should be used in their original form.

Ordinal characteristic

“Qualitative characteristic whose values have a quantitative relation to each other.”

EXAMPLES:

- *The characteristic "clothing size" with the values XXS, XS, S, L, XL, XXL, ..., 7XL,*
- *The characteristic "(European) shoe size" with the values 14, 15, 16, ..., 48, 49.*

The values of these characteristics can be classified and differentiated, e.g. using the relationships “smaller than”, “is equal” or “greater than”.



2 Taking samples

2.1 Randomness of samples

Chapter 3.1 (flipping coins and Example 3) shows how one can draw a conclusion based on a sample from a finite or infinite population about the composition of this population. Such conclusions are an application of inductive statistics. The application is only permitted and sensible when the requirements of independence and randomness of the samples have been fulfilled.

In this context independence means that the result of a sample is not influenced by the result from a previous sample.

Random means that all of the parts in the population being considered have the same chance to make it into the sample (example: drawing lottery numbers).

If these requirements are not met, one runs the risk of drawing false conclusions from these samples, because they are not representative of the population being studied.

2.2 Random numbers

There are several ways to ensure that the selection of parts from a population is random.

If, for example, 10 parts are to be randomly selected from a population of 80 parts, one could number the parts and create a sort of lottery with 80 equally sized tickets and number them 1 to 80, then draw ten tickets. In this case the mechanical mixing of the tickets would guarantee the randomness of the sample.

Another option would be to use a dodecahedron (an platonic solid composed of twelve regular pentagonal faces) and number 10 of the twelve sides from 0 to 9. This is a method similar to "rolling dice" and will help to select a number between 0 and 9 (if the result is an "empty side", simply "roll again").

Due to the object's symmetry, each of the numbers 0 to 9 have the same chance (namely, $1/12$) of being "rolled". By combining consecutive numbers, one could generate desired numbers or cover a number range corresponding to the population.

It is obvious that generating random numbers using "mechanical devices" is relatively cumbersome and time-consuming.

In practice, one can take random numbers directly from a table (see Chapter 2.2.1 and the Appendix) or create them using a pseudorandom number generator on a computer.



2.2.1 Using the random number table

The units from the observed population are numbered in series. It is not necessary to number the units if they are already ordered systematically. The unique mapping of the numbers to the observed units has to be guaranteed.

One chooses a random number from the table (in the Appendix) as a starting number and reads progressively through the number groups to the left or right of this number. These groups can consist of 1, 2, 3, ... , n numbers depending on the size of the population.

Each of the numbers found in this way corresponds to one unit of the population. If one of the random numbers exceeds the size of the population, it is to be omitted. A number is also to be omitted if it is already contained once in the list of numbers.

If the population is already divided into several groups of equal size, it is usually recommendable to first randomly select a group and then randomly a part within this group.

This process is complete if a sequence of random numbers corresponding to the planned sample size has been determined.

EXAMPLE 1:

A sample of ten pumps is to be drawn from a lot consisting of 95 injection pumps. Since the pumps are located on a shelf in the order in which they were produced, it is easiest to assign each of them a consecutive number and select ten numbers randomly using the random numbers table.

To do so, one starts with a random starting number in the random numbers table (see the Appendix), for example with the number 7 at the "intersection" of column 15 and row 35 of the table. One then continues to the right in row 35 taking paired groups:

75 96 04 32 53 01 **99** 35 49 40 **75** 16

The numbers 75 and 99 are omitted because they either exceed the number of parts (99) or have already been taken (75).

The corresponding pumps will be taken from the lot using the numbers determined in this way.

EXAMPLE 2:

A sample of 30 castings is to be taken from a lot of 900.

The castings are in 100 stacks of 9 each. It is therefore beneficial to use the group selection system. For example, begin the selection process with the number "9" at the "intersection" of row 15 with column 21. Now continue, e.g. from left to right and read numbers in pairs of two and combine them with the following number. The first number of each number combination is the respective stack, and the second number is the combination of the running number of the parts in the stack, which may be numbered from top to bottom.

94/2	13/7	75/6	40/1	28/7	99/4
50/1	82/1	25/8	03/6	06/6	09/9
94/2	00/0	60/8	70/1	76/3	40/7
70/8	15/4	30/4	57/4	45/4	80/0
56/1	88/4	4			

continue reading from column 0 in row 16:



44/4 37/8 19/2 **87/0** 51/1 60/6
89/9 24/1

Note on the bold numbers:

- Number 00 corresponds to stack 100.
- The numbers 00/0, 80/0 and 87/0 are omitted since part 0 is not included in a stack.
- The second occurrence of number 94/2 is omitted.

2.2.2 Instructions for use

It is especially beneficial to use random numbers when the lot to be examined is already systematically ordered, e.g. parts stored on a shelf, parts numbered during production or parts sorted into storage containers.

With larger quantities, it is recommended to divide the parts into smaller groups and then take samples from these.

If the lot to be examined has already been completely mixed during production or storage, one can forgo using random numbers. In this case, the sample must be taken in such a way that each part has a chance of making it into the sample.



3 Randomness and probability

3.1 The concept of probability

The words “probability” or “probable” come up often in everyday conversation, in which one wants to speculate about a situation or make a prediction on events, e.g.:

“It is very probable that the football team X will win the game against team Y on Saturday.”

“My car is only almost six months old; it is unlikely that it will break down because of any technical issues during my holiday trip next week.”

“I would rather take the train to Munich instead of driving, I will definitely get there before 10 a.m.”

In everyday speech, the term “probable” is often replaced with other terms like “impossible”, “maybe”, “likely”, “pretty certain”, or “dead sure”, which, in our experience, are supposed to represent a way to trust the correctness of a statement. However, depending on the person using such a term, their mood (euphoric, depressed) and the respective situation, each term might have a completely different meaning.

On the other hand, mathematical probability is a number that is closely related to the results of a random experiment.

“Random experiment” designates a process that can be repeated as often as desired and whose results are not predictable.

A classic random experiment that is almost always discussed in textbooks about calculating probability is flipping a coin. This random experiment is usually conducted before a ballgame in order to decide who chooses which side on the playing field.

Obviously, both teams accept that the results from flipping a coin are impossible to predict and that, based on the (sufficient) symmetry of the coin, the results of “heads” and “tails” are just as probable.

According to the classic definition, the mathematical probability $P(A)$ of result A in a random experiment is given by

$$P(A) = \frac{g}{m}.$$

Where

g is the number of (favorable) cases where A occurs and

m is the number of all possible cases

in the experiment at hand.

With regard to flipping a coin, this means:

The probability of the event “head” is $P(\text{“head”}) = \frac{1}{2} = 0.5 = 50\%$.

The number g of cases where “heads” occurs (favorable result for a team X if they chose “heads”) is equal to 1, the number of all possible outcomes of the coin toss is equal to 2 (“heads” and “tails”).

The same also applies for team Y: $P(\text{“tails”}) = \frac{1}{2} = 0.5 = 50\%$.



The number g of cases where “tails” occurs (favorable result for a team since they inevitably chose “tails”) is equal to 1, the number of all possible outcomes of the coin toss is equal to 2 (“heads” and “tails”).

The “symmetry of probability” obviously results from the symmetry of the coin. Both results are just as probable: $P(\text{“heads”}) = P(\text{“tails”}) = \frac{1}{2}$.

A predictable probability like this is referred to as mathematic or “a priori” probability (Latin: a priori = “in the first place”).

Applying this idea to a regular die, one can predict that the probability of rolling a “6” is $P(\text{“6”}) = \frac{1}{6}$, because

$g = 1 = \text{number of favorable results (6) and}$

$m = 6 = \text{Number of all possible results (1, 2, 3, 4, 5, 6)}$

in this experiment.

So it is not necessary to actually use a die or coin to conduct random experiments in order to draw a conclusion about the probability of the results.

If one applies the above definition of probability to other random experiments, it is clear that the probability P is a number between zero and one.

NOTE:

It should be mentioned here that there is also an axiomatic approach to the concept of probability, where the probability P of an event is defined as a non-negative real number that can equal, but not exceed one. Moreover, there exist rules associated with probabilities, which we will explain using the following examples.

The sum of the individual probabilities for the results of a random experiment is always equal to one:

$$\text{Coin: } P(\text{“heads”}) + P(\text{“tails”}) = \frac{1}{2} + \frac{1}{2} = 1$$

$$\text{Dice: } P(\text{“1”}) + P(\text{“2”}) + \dots + P(\text{“6”}) = \frac{1}{6} + \frac{1}{6} + \dots + \frac{1}{6} = 1.$$

An event that always occurs during an experiment is called a “certain event” for this experiment. For example, when rolling dice, one of the numbers 1, 2, 3, 4, 5, 6 is a certain event with the probability $P(\text{“1” or “2” or ... or “6”}) = 1$.

An event that never occurs during an experiment is called an “impossible event” for this experiment. For example, when rolling a die, the number 7 is an impossible event with the probability $P(\text{“7”}) = 0$.

Until now, we have assumed that the subject of our considerations is physically known or can easily be approximated, and it was shown that the results of random experiments with this subject (coin, dice) cannot be predicted individually, but can be represented as a population. In particular, probabilities can be determined for the individual results without even conducting the experiments.

The situation changes if we, for example, consider a die that seems completely geometrically flawless that possesses an inhomogeneous inner distribution of mass that makes the die “roll” the number 6 more often than the other numbers. How can one determine the probability of rolling the result “6” in this case?



This is obviously only possible by rolling the die many times (e.g. 100), noting the results using a tally chart and counting how often each number actually occurs. The number of 6-rolls compared to the total number of rolls is a suitable measure for the probability of the occurrence of the result “6”.

This process allows one to determine a probability for an experiment that was actually conducted. One draws a conclusion about the underlying (unknown) probability in the experiment based on the observed results. This type of probability is also called an “a posteriori” probability (Latin: a posteriori = “in hindsight”).

If one repeats the entire experiment (100 rolls), the results will not necessarily show the same number of 6-rolls like the first time. The experimentally determined probability $P(\text{“6”})$ can therefore only be an estimation of the unknown probability P of achieving a “6” with this die. This estimation is better the larger the number of times the die has been rolled. Mathematically, this phenomenon is described by the “law of large numbers”.

If one calculates the relative frequency of the result “6” by consecutively rolling a regular die, the results will show that this quantity is getting closer to the theoretical value

$$P(\text{“6”}) = \frac{1}{6} = 16.\bar{6} \% \text{ (see Table 3.1 and Figure 3.1).}$$

Roll No.	Roll result	rel. frequency of the result “6”	Roll No.	Roll result	rel. frequency of the result “6”
1	5	0.0	492	2	16.9
2	2	0.0	493	1	16.8
3	4	0.0	494	3	16.8
4	6	25.0	495	6	17.0
5	5	20.0	496	2	16.9
6	3	16.7	497	4	16.9
7	3	14.3	498	5	16.9
8	6	25.0	499	3	16.8
9	3	22.2	500	4	16.8

Table 3.1



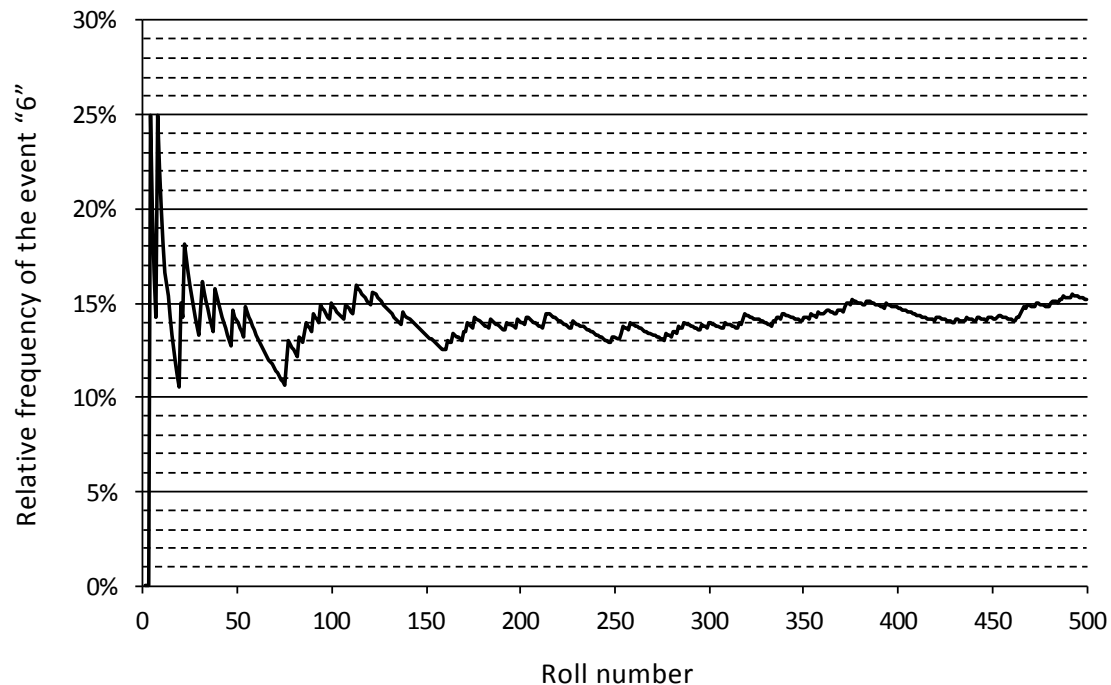


Fig. 3.1: Illustration of the law of large numbers. The relative frequency of the result “6”, when rolling a die, approaches the theoretic value of 16.6%, if sufficiently many rolls have been performed.

This law of limits is important with regard to the possibility of estimating the fraction of non-conforming parts in a population (e.g. production quantity) based on a sample. Sampling (with replacement) is, in the above terminology, a random experiment, which can be conducted as often as desired. We will continue this thought in the following example.

EXAMPLE 3:

A box contains a total of 4000 balls. One portion of them is black, the rest are white. If one tasks several people to take a look into the box and estimate the ratio p of black balls, experience shows that the number will be between 10% and 40%.

The problem is now to estimate the actual proportion p of black balls by sampling.

10 samples of 50 balls each are taken. This process can be demonstrated quite well if one uses a metal plate with 50 grooves. The ratio of black balls in the sample is determined and noted after taking each sample. Subsequently, the plate is completely emptied into the box and the balls are mixed.

The following table gives the result of such an experiment.



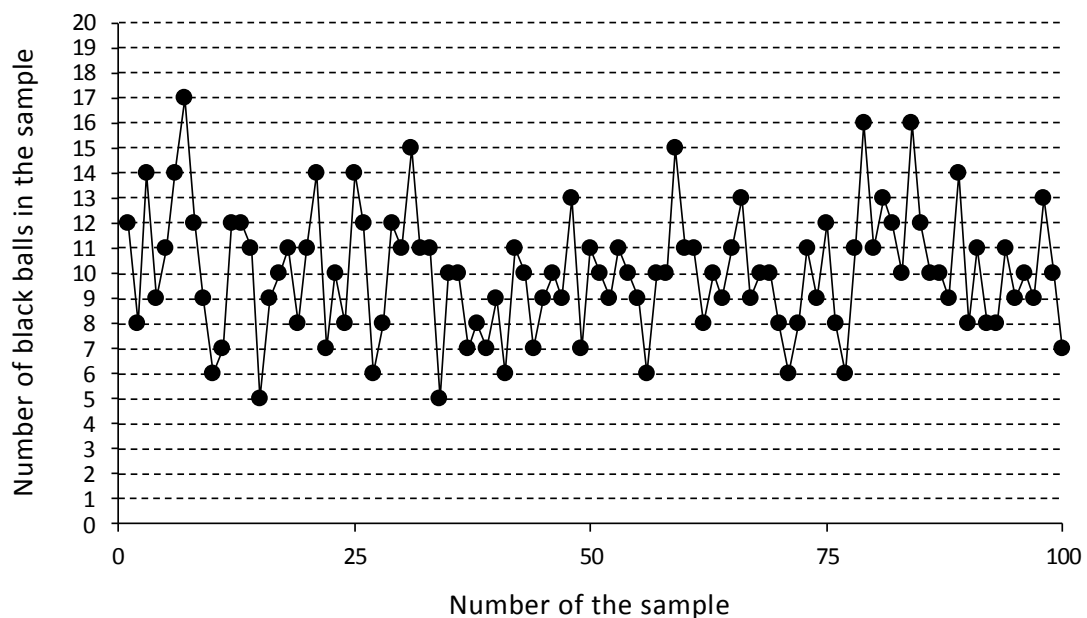
Sample no.	black balls		white balls	
	Number	Proportion	Number	Proportion
1	10	20%	40	80%
2	10	20%	40	80%
3	11	22%	39	78%
4	10	20%	40	80%
5	12	24%	38	76%
6	10	20%	40	80%
7	14	28%	36	72%
8	11	22%	39	78%
9	8	16%	42	84%
10	9	18%	41	82%
Mean	10.5	21%	39.5	79%

Table 3.2

It is obvious that the number of black balls found can differ from sample to sample, but they produce a mean of about 10 or 11 black balls, corresponding to a ratio of about 20% or 21%.

If only one sample is taken, and eight balls are found as in case no. 9, one would have to assume or the time being, due to the lack of additional information, that the estimation $p = 16\%$ is not too far from the actual value.

Both the table above and the following illustration showing the results of 100 samples make it clear that, with repeated samples, a certain range of dispersion arises for the number of black balls and the proportion p in the sample.

**Fig. 3.2:** Results of 100 samples with 50 balls each (with replacement)

With the help of a tally chart, one can count how often respectively 1, 2, 3, ..., 49, 50 black balls were found in a sample. The frequency of each result and the associated relative frequency (frequency/100) are given in the following table.

<i>Result: Number i of black balls in the sample</i>	<i>Frequency of the result i</i>	<i>relative frequency of the result i</i>
<5	0	0%
5	2	2%
6	6	6%
7	7	7%
8	12	12%
9	14	14%
10	18	18%
11	18	18%
12	9	9%
13	4	4%
14	5	5%
15	2	2%
16	2	2%
17	1	1%
18	0	0%
>18	0	0%

Table 3.3

The values in the table can be represented using a bar graph:

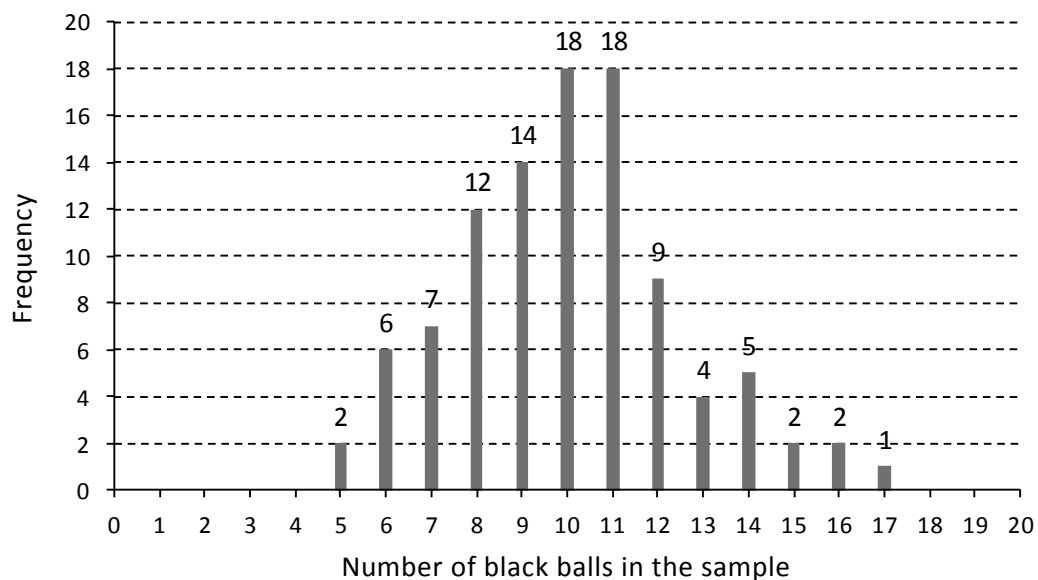


Fig. 3.3: Bar graph to represent 100 sample results



Figure 3.3 shows that sample results of a very large or very small number of black balls rarely or “practically never” occur and that about 10 or 11 black balls per sample occur very often. 10 (11) black balls in a sample of size $n=50$ correspond to 20% (21%). Based on the sample results and Figure 3.3, one would also be able to estimate the unknown ratio p at about 20% to 21%.

Such estimations are an application of the field of inductive statistics. Based on the results of one sample, one can estimate a parameter (here the proportion p of black balls) of the total population (all balls in the box).

With the example described here, we are lucky to know the true ratio p of black balls in the finite population. The box contains 800 black balls and 3200 white balls; the ratio of black balls is therefore $p = 20\%$.

We get back to this example again in Chapter 3.3.1 and will consider then how to explain the results from Figure 3.3.

3.2 Calculation rules for probabilities

As we saw in Chapter 3.1, the probability of seeing the result “heads” when flipping a coin is equal to $1/2$.

We will now ask ourselves, what is the probability of getting “heads” in three consecutive coin tosses?

A simple consideration leads quickly to the solution of this problem. There are a total of eight possible results of this experiment (“H” stands for heads, “T” stands for tails):

1. Possibility: HHH
2. Possibility: HHT
3. Possibility: HTH
4. Possibility: THH
5. Possibility: TTH
6. Possibility: THT
7. Possibility: HTT
8. Possibility: TTT

Only one of the eight results is “HHH”.

According to the definition of mathematical probability in Chapter 3.1, the probability of this result is:

$$P(\text{“HHH”}) = \frac{\text{Number of favorable outcomes}}{\text{Number of all possible outcomes}} = \frac{g}{m} = \frac{1}{8}.$$

The word “favorable” is to be understood in the sense of game theory. The result “HHH” is favorable for the player if he, for example, bet an amount of money on exactly this result.

The probability $P(\text{“HHH”}) = \frac{1}{8}$ can, however, also be calculated from the individual probabilities of the individual coin tosses.

Since the coin does not have a “memory” and knows nothing about the results of previous tosses (provided that the player cannot influence the result), coin tossing is an independent event. According to the multiplication rule for probabilities, the probability for a



series of such independent events can be calculated as the product of the respective individual probabilities:

$$P(\text{"HHH"}) = P(\text{"H"}) \cdot P(\text{"H"}) \cdot P(\text{"H"}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}.$$

How great is the probability that "heads" occurs twice and "tails" once with three coin tosses? The order of the individual results is not important for this question.

According to the multiplication rule, the three possibilities "HHT", "HTH" and "THH" occur with the same probability:

$$P(\text{"HHT"}) = P(\text{"H"}) \cdot P(\text{"H"}) \cdot P(\text{"T"}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8},$$

$$P(\text{"HTH"}) = P(\text{"H"}) \cdot P(\text{"T"}) \cdot P(\text{"H"}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8},$$

$$P(\text{"THH"}) = P(\text{"T"}) \cdot P(\text{"H"}) \cdot P(\text{"H"}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}.$$

However, only one of the three possibilities can occur when the coin is tossed three times, either "HHT" or "HTH" or "THH". The individual possibilities are mutually exclusive.

The total probability for the result of two "heads" and one "tails" can then be calculated according to the addition rule of mutually exclusive probabilities:

$$P(\text{"HHT" or "HTH" or "THH"}) = P(\text{"HHT"}) + P(\text{"HTH"}) + P(\text{"THH"}) = \frac{3}{8}$$

In the same way, one can calculate the total probability for the result of one "heads" and two "tails":

$$P(\text{"HTT" or "THT" or "TTH"}) = P(\text{"HTT"}) + P(\text{"THT"}) + P(\text{"TTH"}) = \frac{3}{8}$$

Finally, one obtains the probability for the result "TTT" similar to "HHH", according to the multiplication rule:

$$P(\text{"TTT"}) = P(\text{"T"}) \cdot P(\text{"T"}) \cdot P(\text{"T"}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}.$$

The probability that one of the eight results mentioned on the previous page occurs (certain event) is equal to the sum of the probabilities calculated above:

$$\begin{aligned} P(\text{"HHH" or "HTH" or ... or "TTT"}) &= \\ &= P(\text{"HHH"}) + P(\text{"HHT"}) + \dots + P(\text{"TTT"}) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1. \end{aligned}$$



3.3 Probability distributions

The probabilities for the four possibilities calculated in Chapter 3.2

no “heads”, three times “tails”,

one time “heads”, two times “tails”,

two times “heads”, one time “tails”,

three times “heads”, no “tails”,

can be represented as follows:

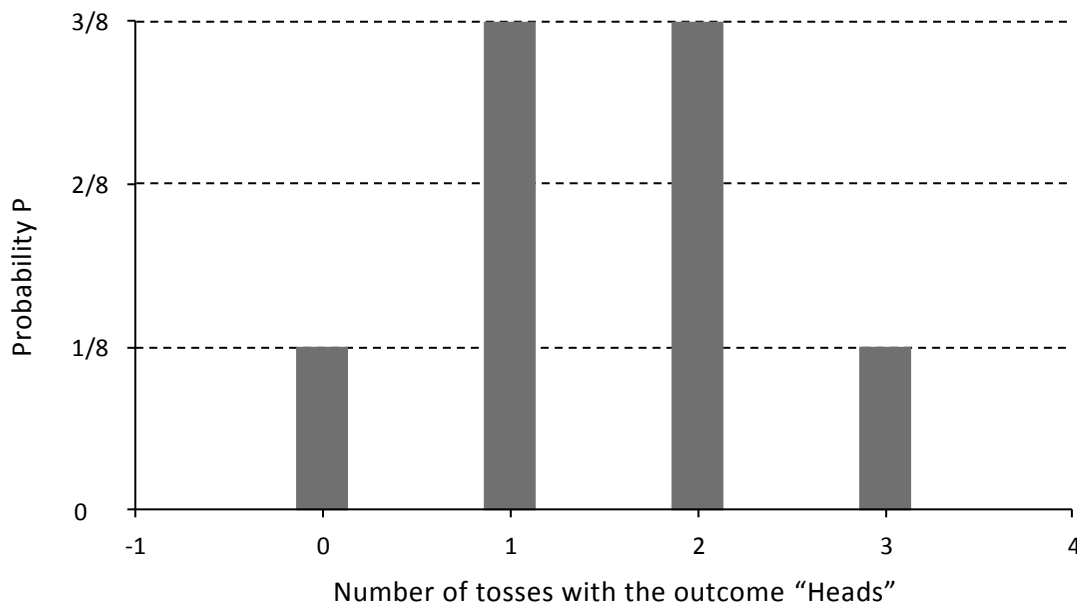


Fig. 3.4: Illustration of the probability function for three coin tosses

This is a depiction of the probability function of the individual results of the random experiment described above.

If one wants to be mathematically exact, one would have to say that the probability function is defined for a certain quantity, the event space. The event space corresponds in this case to the quantity of the four possibilities previously mentioned. The probability function assigns every element of this quantity a real number, the probability of the event.

The vertical columns in Figure 3.4 are only drawn that wide for illustrative purposes. Strictly speaking, the probability function only assumes the values $1/8$, $3/8$, $3/8$ and $1/8$ at the respective positions 0, 1, 2 and 3. It is zero for all the other positions on the number line.

3.3.1 Binomial distribution

The probability function for tossing a coin four times can be calculated in a manner similar to the calculation of the probability function for tossing a coin three times. We do not explicitly want to do that here and merely present an illustration of this function. It assumes the respective accompanying values $1/16$, $4/16$, $6/16$, $4/16$, $1/16$ for the events no “heads”, one time “heads”, ..., four times “heads”.



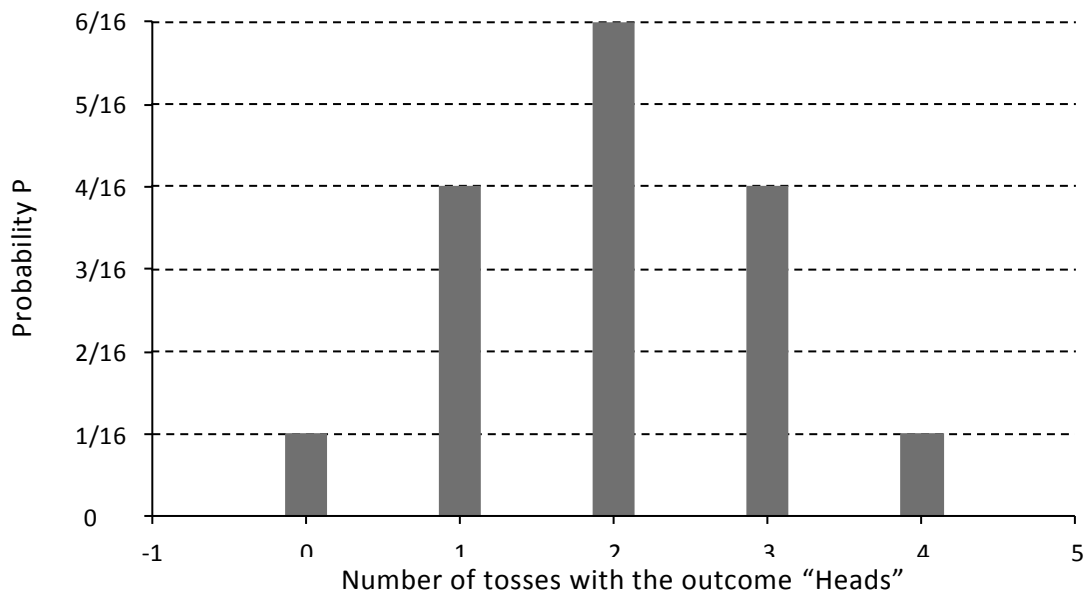


Fig. 3.5: Illustration of the probability function for the four time coin toss

Figures 3.4 and 3.5 show a striking symmetry for the probability function, which is maintained in the generalization to the n -time coin toss. In the following, we will see that this fact can be explained using a simple law of deduction.

EXAMPLE 4:

From a lot of housing bases with threaded holes, one part will be randomly chosen and compared to boundary samples. If the part conforms to specifications, it will be designated with good. If it does not (e.g. if the threading is missing), it will be designated with bad. After noting the results of the examination, the part will be returned to the lot.

If one removes two parts, the result might read:

Part 1 good,	part 2 good	or
Part 1 good,	part 2 bad	or
Part 1 bad,	part 2 good	or
Part 1 bad,	part 2 bad.	

This corresponds to a situation with two coin flips, if one considers "good" as "heads" and "bad" as "tails". In contrast to a coin flip, the probability for both results when chosen for screening is not known.

In the following, we designate the probability for the occurrence of a bad part with p' and the probability for the occurrence of a good part with $q'=1-p'$. According to the multiplication rule for independent results, one obtains the following probability for the four possible results:



<i>Examination result</i>	<i>Probability</i>
<i>good - good</i>	$q' \cdot q'$
<i>good - bad</i>	$q' \cdot p'$
<i>bad - good</i>	$p' \cdot q'$
<i>bad - bad</i>	$p' \cdot p'$

According to the summation rule for mutually exclusive results, the total probability is

$$P = p' \cdot p' + p' \cdot q' + q' \cdot p' + q' \cdot q' = p'^2 + 2 \cdot p' \cdot q' + q'^2 = (p' + q')^2 = 1.$$

The coefficients in the expression $1 \cdot p^2 + 2 \cdot p \cdot q + 1 \cdot q^2$ are called the binomial coefficients and can be determined using Pascal's triangle:

[illegible]

Every number except for 1 can be calculated by summing the numbers diagonally above to the left and right of its position. This triangle can be continued downward as far as desired.

Binomial coefficients provide a simple way to calculate the probability of an individual result when taking three, four, ... parts from a lot. For example, when taking four parts:

$$(p' + q')^4 = p'^4 + 4 \cdot p'^3 \cdot q' + 6 \cdot p'^2 \cdot q'^2 + 4 \cdot p' \cdot q'^3 + q'^4 = 1.$$

In a sample of size 4, the probability of

finding 4 bad parts is p'^4

finding 3 bad parts is $4 \cdot p^3 \cdot q$

finding 2 bad parts is $6 \cdot p'^2 \cdot q'^2$

finding 1 bad part is $4 \cdot p' \cdot q'^3$

finding no bad part is q^4 .



The binomial coefficients can be written concisely as $\binom{n}{i}$ (say: n over i) and calculated using the formula $\binom{n}{i} = \frac{n!}{i! \cdot (n-i)!}$.

$n!$ (say: n factorial) means $n! = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 3 \cdot 2 \cdot 1$.

For example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.

The following rules apply when calculating with binomial coefficients:

$$\binom{n}{n-1} = \binom{n}{1} = n \quad \text{and} \quad \binom{n}{0} = \binom{n}{n} = 1.$$

The probability that in a sample of size n there are exactly i non-conforming parts is

$$P_{n,i} = \binom{n}{i} \cdot p^i \cdot (1-p)^{n-i}.$$

$P_{n,i}$ is the probability function of the binomial distribution.

The binomial distribution can be used without limitations if the total population is infinitely large (fictitious population) or, more realistically, if the sample size measured against the real population is insignificantly small or the parts sampled are always returned to the population.

In this regard we want to turn back to Example 3 from Chapter 3.1. We were looking at a box with 4000 balls with a proportion of black balls of $p' = 20\%$ and a proportion of white balls of $q' = 1 - p' = 80\%$. Since the balls sampled will be placed back into the box, the binomial distribution can be used for this problem.

Here the number of balls found in a sample of $n=50$ is subject to the binomial distribution with the probability function:

$$P_{50,i} = \binom{50}{i} \cdot 0.2^i \cdot (1-0.2)^{50-i}.$$

$P_{50,i}$ is the probability to find $n=50$ exactly i black balls in a sample of size.

The following table contains several values for $P_{50,i}$. Figure 3.6 illustrates these values clearly. If one compares Tables 3.3 and 3.4 or Figures 3.3 and 3.6, it is clear that the distribution determined experimentally in the demonstration from Chapter 3.1 approximates the theoretical distribution.



i	$P_{50,i} / \%$	$\sum_{k=0}^i P_{50,k} / \%$
0	0.0	0.0
1	0.0	0.0
2	0.1	0.1
3	0.4	0.6
4	1.3	1.8
5	3.0	4.8
6	5.5	10.3
7	8.7	19.0
8	11.7	30.7
9	13.6	44.4
10	14.0	58.4
11	12.7	71.1
12	10.3	81.4
13	7.5	88.9
14	5.0	93.9
15	3.0	96.9
16	1.6	98.6
17	0.8	99.4
18	0.4	99.7
19	0.2	99.9
20	0.1	99.97

Table 3.4:

Values of the probability function and the distribution function of the binomial distribution with the parameters $p' = 20\%$ and $n = 50$

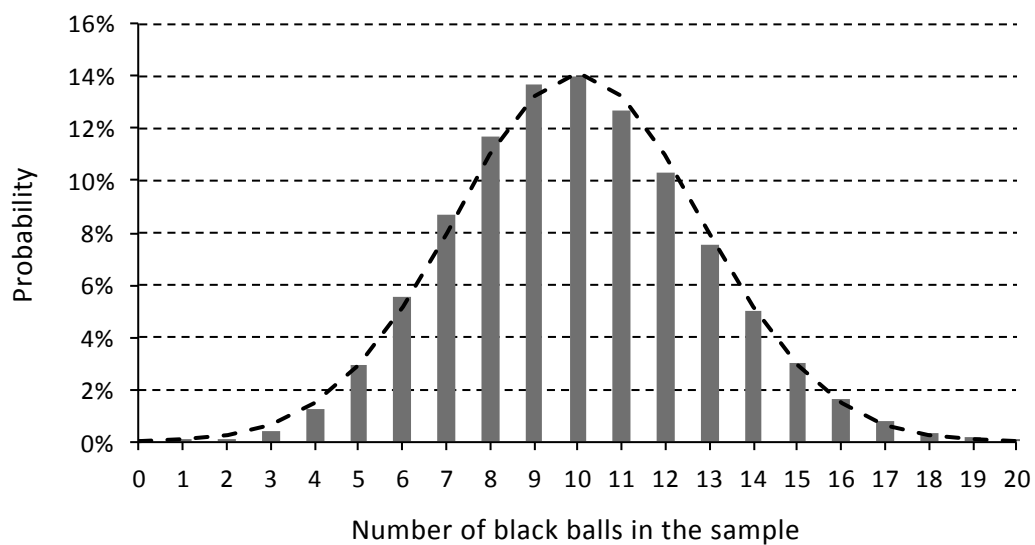


Fig. 3.6: Schematic of the probability function of the binomial distribution with the parameters $p' = 20\%$ and $n = 50$ (dashed: probability density function of the normal distribution)



The values of the probability function $P_{50,i}$ and the distribution function $\sum_{k=0}^i P_{50,k}$, which belongs to it, are given in the third column of Table 3.4.

The distribution function gives, in this example, the probability of finding at most i black balls in the sample. The summation symbol means that the individual probabilities for the results 0, 1, 2, ... i black balls must be added up:

$$\sum_{k=0}^i P_{50,k} = P_{50,0} + P_{50,1} + \dots + P_{50,i}.$$

Accordingly, each number in the third column (except for the rounding errors) is the result of adding the values in the second column up to this line. The distribution function is shown in Figure 3.7. From Table 3.4 and Figure 3.7, one can see that in approximately 2% of all cases $i \leq 4$ and in approximately 97% of all other cases $i \leq 15$, black balls are included in a sample. Or, said differently, one finds at least four and no more than 15 black balls in the sample in about 95% of the cases.

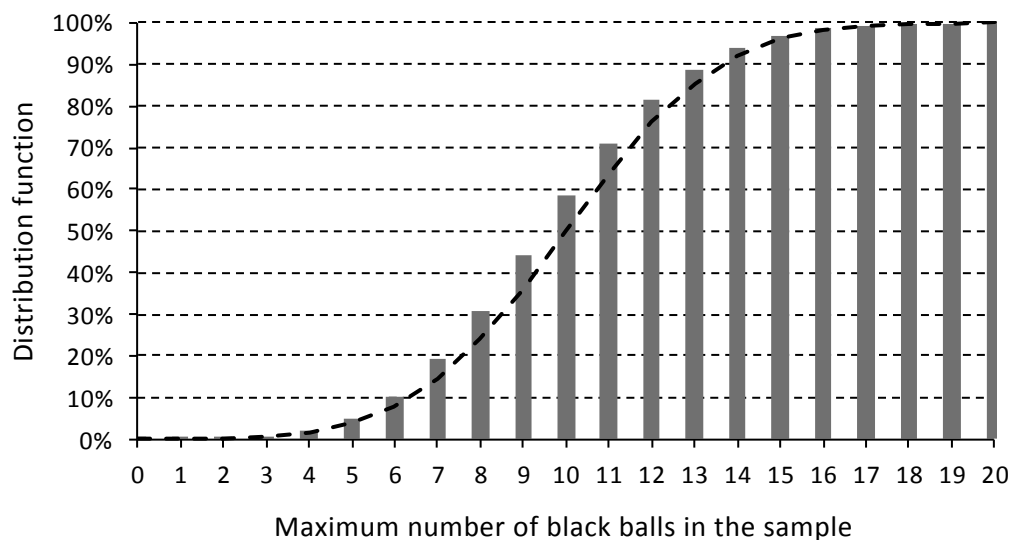


Fig. 3.7: Schematic illustration of the binomial distribution function with the parameters $p=20\%$ and $n=50$ (dashed: normal distribution)

All of the values up to $i=20$ are provided in Table 3.4 and Figures 3.6 and 3.7. The probability function is almost always zero for $i > 20$, the distribution function is almost always one (100%). For comparison purposes, the probability density function or the normal distribution function with the mean $\mu=10$ and standard deviation $\sigma=2.8$ is displayed as a solid line in the graphs. It is apparent that the binomial distribution in this example is a very good approximation of the normal distribution. We will come back to this in Chapter 4.1.

3.3.2 Hypergeometric distribution

In order to use the binomial distribution it was a prerequisite in Chapter 3.3.1. that the population has to stay the same from sample to sample. For a finite population this is accomplished by returning each preceding sample to the case. If the sampled parts are not put back, the population changes from sample to sample. The parts already taken will not be contained in the following samples, and the probability for the remaining parts in the total population to be "drawn" next increases with every sample. The hypergeometric distribution takes this fact into account. The probability function of this distribution is:



$$P_{N,n,i} = \frac{\binom{N \cdot p'}{i} \cdot \binom{N \cdot (1-p')}{n-i}}{\binom{N}{n}}.$$

Here, N designates the size of the population, i.e. the total number of all parts in a given lot. It is not easy to calculate this expression even by using a computer due to the large numbers resulting from the possible factorials (e.g. $1000!$). Moreover, depending on the size of N , n and p' , factorials for non-integer rational numbers may arise, which can only be calculated using the gamma function.

In practice, the population is usually considerably larger than the sample taken ($N \gg n$), meaning the binomial distribution can be used by approximation.

3.3.3 Poisson distribution

If one knows that the interesting event, e.g. finding a non-conforming part in a sample of size $n=1$, rarely occurs, while the number of executions of the random experiments "drawing a sample of size $n=1$ " is very large, the Poisson distribution can help to approximate the quite unwieldy binomial distribution caused by the binomial coefficients.

In mathematical terms, the probability function of the Poisson distribution produces

$$P_{n,i} = \frac{(n \cdot p')^i}{i!} \cdot e^{-n \cdot p'}$$

as a passage to the limit of the probability function of the binomial distribution, when n approaches infinity and p' approaches zero, while the product $n \cdot p'$ remains constant.

The mean of the Poisson distribution is $\mu = n \cdot p'$, its standard deviation is $\sigma = \sqrt{\mu} = \sqrt{n \cdot p'}$.

Calculation example: The value given Table 3.4 for the probability of finding $i = 10$ black balls in the sample of size $n = 50$ is 0.14 or 14%. Using the probability function of the Poisson distribution yields

$$P_{50,10} = \frac{(50 \cdot 0.2)^{10}}{10!} \cdot e^{-50 \cdot 0.2} = \frac{10^{10}}{10!} \cdot e^{-10} = 0.125 = 12.5\%.$$

3.3.4 Valid ranges of discrete distributions

We combine essential properties of the above described distributions in the following table.



Distribution	Valid	Mean μ	Standard deviation σ	Probability function P
Hyper-geometric distribution	generally	$n \cdot p'$	$\sqrt{n \cdot p' \cdot (1-p') \cdot (1-\frac{n}{N})}$	$\frac{\binom{N \cdot p'}{i} \cdot \binom{N \cdot (1-p')}{n-i}}{\binom{N}{n}}$
Binomial-distribution	$N \rightarrow \infty$ or sampling with replacing	$n \cdot p'$	$\sqrt{n \cdot p' \cdot (1-p')}$	$\binom{n}{i} \cdot p'^i \cdot (1-p')^{n-i}$
Poisson-distribution	approximately for $n \leq 0.1 \cdot N$ and $n \cdot p' \geq 4$	$n \cdot p'$	$\sqrt{\mu} = \sqrt{n \cdot p'}$	$\frac{(n \cdot p')^i}{i!} \cdot e^{-n \cdot p'}$
<p>$n!$ (say: n factorial) means $n! = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 3 \cdot 2 \cdot 1$.</p> <p>$\binom{n}{i}$ (say: n over i) is calculated with the formula</p> $\binom{n}{i} = \frac{n!}{i! \cdot (n-i)!} = \frac{n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot (n-i+1)}{i \cdot (i-1) \cdot (i-2) \cdot \dots \cdot 3 \cdot 2 \cdot 1}.$				

Table 3.5

$P_{n,i}$ is the probability that, in a sample of size n from a population of size N , exactly i non-conforming parts will be found when the ratio of non-conforming parts in the population is p' .

3.3.5 Approximation approaches with discrete distributions

The Poisson distribution is given by the binomial distribution at the passage to the limit, where p' approaches zero and n approaches infinity. The quantity p' refers to the population.

According to a limit theorem of statistics, the binomial distribution becomes the normal distribution for sufficiently large n and a given p' with $0 < p' < 1$.

Of course, in practice the sample size n will not become infinitely large. However, it is possible, even for finite values of n , to approximate the binomial distribution via the Poisson distribution, and the normal distribution or the Poisson distribution via the normal distribution. Various details on the conditions of these approximations are found in the literature (e.g. [7] and [8]), however, for practical use these can be considered the same. These conditions are commonly summarized in a diagram (Figure 3.8) that provides information on which combination of n and p' the binomial distribution can be substituted by the Poisson distribution or the normal distribution (see for example [7] and [20]).



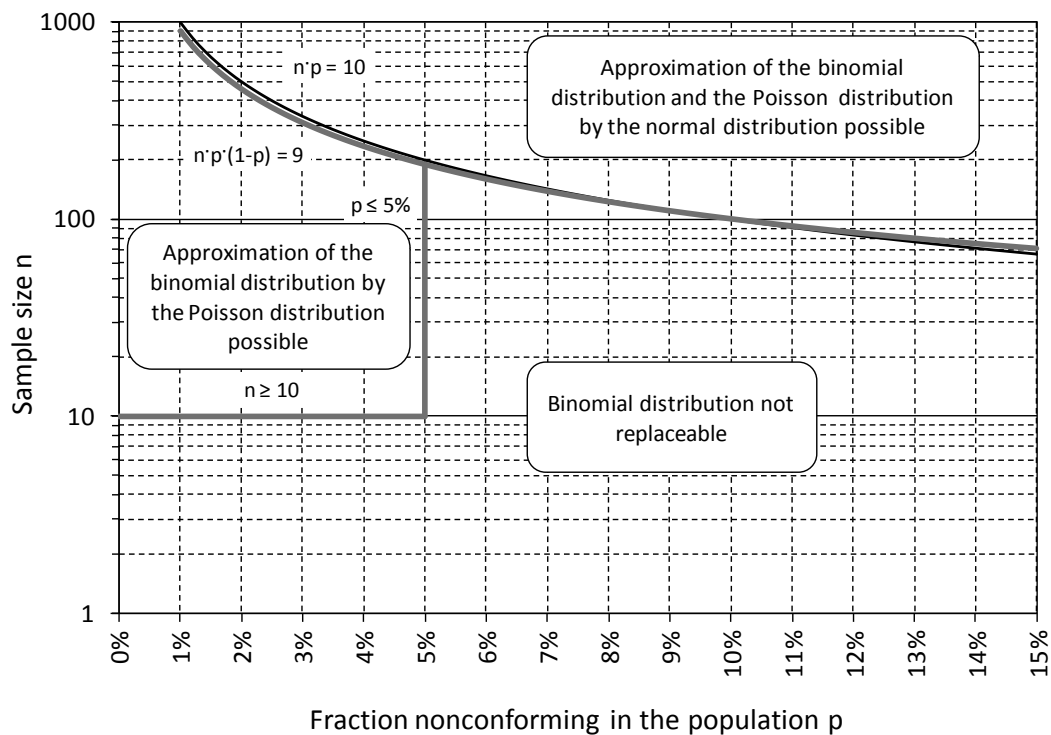


Fig. 3.8: Formal replacement of the binomial distribution by the Poisson or the normal distribution.

Approximation of the binomial distribution by the normal distribution

Figures 3.6 and 3.7 clearly show that the normal distribution approximates a binomial distribution very well with the example considered there. This approximation is always possible if $n \cdot p' \geq 10$ (see [7]). The mean and the standard deviation of the normal distribution are then $\mu = n \cdot p'$ and $\sigma = \sqrt{n \cdot p' \cdot (1-p')}$.

This approximation method is used in Chapter 4.1 to determine in an easy manner the control limits of an np-chart.

Approximation of the binomial distribution by the Poisson distribution

A binomial distribution with large sample size n and small p' (e.g. number of non-conforming parts in the population) can be approximated by a Poisson distribution with $\mu = n \cdot p'$. A large sample size and a small p' mean $n > 10$ and $p' < 0.05$ according to [8].

The approximation of the binomial distribution by the Poisson distribution is used in Chapter 5.2 to easily calculate the control probabilities with regard to the simple sampling scheme.

Approximation of the Poisson distribution by the normal distribution

For $n \cdot p' \geq 10$ the values of the Poisson distribution conform very well with the values of the normal distribution with $\mu = n \cdot p'$ and $\sigma = \sqrt{\mu} = \sqrt{n \cdot p'}$. We use this fact in Chapter 4.3 to calculate the control limits of the c-chart.



4 Applications of Discrete Distributions within the Scope of Statistical Process Control (SPC)

At the end of Chapter 3.3.1, in the example under consideration, it was determined that in about 95% of all cases between 4 and 15 black balls were to be expected in the sample.

What conclusion should one draw if, against all expectations, for example 25 black balls had been contained in the sample? Well, one could of course assume that this is one of the rarest cases that could arise, statistically speaking. One could, however, suppose that the ratio of black balls in the box had increased considerably since the last probe, for example to 50%, because then the observed event would not be unusual, but rather be an expected normal case.

NOTE:

The second guess for an observer to this example would only be obvious if the box is not visible between samplings, of course, giving rise to a chance for manipulation.

The thought process described above creates the basis for the use of discrete distributions in Statistical Process Control (SPC).

One takes samples from a production process at regular intervals and determines, for example, the number of non-conforming parts contained within. This number is noted on the form, the control chart, and entered into a diagram on this control chart (similar to Figure 4.1).

If the respective number found lies within the random variation range bordered by solid lines on the control chart (the control limits), one can assume that the current process still corresponds to the usual conditions. However, if, for example, the upper control limit is exceeded, one can conclude that the process conditions have deteriorated, meaning the fraction non-conforming has considerably increased. This procedure corresponds to a statistical conclusion from the sample about the population.

Quality control charts for discrete characteristics (countable characteristics) are based, in contrast to control charts for continuous characteristics that are focused on avoiding non-conformities, on the existence of non-conformities. They are designed to show the user whenever a process status will change with respect to an actual state that cannot be improved any further with justifiable effort. The user would therefore like to ascertain whether the already unsatisfactory process conditions will continue to deteriorate.

The following types of control charts exist for discrete characteristics:

- np-chart
To control the number of non-conforming units with constant sample size
- p-chart
To control the relative number of non-conforming units with constant or variable sample size
- c-chart
To control the number of non-conformities per unit or per unit quantity with constant sample size



- u-chart

To control the number of non-conformities per unit or per unit quantity with constant or variable sample size

The way these charts work, they require a comparably large number of errors, which is unacceptable considering the level of ppm reached today and the goal of zero defects. For this reason, sampling inspection of discrete characteristics is no longer reasonable. Therefore, only a few theoretical basics will be represented here.

4.1 np-chart

With regard to the np- and the p-chart, in the following we will consider a production process where we know that a constant portion p' of the produced parts is non-conforming. This portion p' is generally not known, it rather must be estimated using a series of samples (pre-production). If one draws a sample of size n , one can expect to find about $n \cdot p'$ non-conforming parts within said sample.

If p' is very small, the sample size n must be chosen sufficiently large, so that some of the non-conforming parts will even be included in the sample.

The following is a good rule of thumb for the minimal sample size:

The sample size must be at least large enough that the average number of non-conformities in the sample is greater than or equal to 4 (10 is better). For the np- and p-chart, due to a sample size of $n \geq \frac{400}{\bar{p}}$ results, where \bar{p} is the proportion non-conforming in percent.

When repeating the sampling process with constant sample size n , the result is generally a random number of non-conforming parts per sample that varies around an average value. This average number of non-conforming units can be designated as the mean $n \cdot \bar{p}$ of the non-conforming units in a series of samples.

$$\text{Average value of non-conforming units: } n\bar{p} = \frac{n \cdot p_1 + n \cdot p_2 + \dots + n \cdot p_m}{m} = \frac{1}{m} \cdot \sum_{i=1}^m n \cdot p_i$$

m = number of samples

n = sample size

np_1 = number of non-conforming units in the first sample

np_i = number of non-conforming units in the i -th sample

If one divides the mean $n \cdot \bar{p}$ by the sample size n , one obtains \bar{p} , an estimation value for the share of non-conforming units p' .

The quality control chart for the share of non-conforming parts (np-chart) is based on the binomial distribution (see Chapter 3.3.1). The probability function f of the binomial distribution indicates the probability of finding exactly k non-conforming parts in a sample of size n , if the share of non-conforming parts in the total population produced is p' :

$$f(k; n; p') = \binom{n}{k} \cdot p'^k \cdot (1 - p')^{n-k}.$$

The mean of the binomial distribution is $\mu = n \cdot p'$,



its standard deviation is $\sigma = \sqrt{n \cdot p' \cdot (1 - p')} = \sqrt{n \cdot p' \cdot \left(1 - \frac{n \cdot p'}{n}\right)}$.

If the mean μ of the binomial distribution is sufficiently large (this is guaranteed by a minimum sample size), this formula approximates the normal distribution very well (see Figure 3.6 in Chapter 3.3.1).

The control limits are defined, so that around 99% of the sample results lie within these limits if the proportion non-conforming remains constant. If a sample result lies outside the control limits, one must assume that the proportion non-conforming has changed (i.e. has increased or decreased).

The 99% random variation interval of a normally distributed characteristic is limited by control limits that can be calculated as follows (upper or lower control limits):

$$UCL = \hat{\mu} + 2.58 \cdot \hat{\sigma}$$

$$LCL = \hat{\mu} - 2.58 \cdot \hat{\sigma}$$

The number 2.58 is the two-sided threshold value of the standard normal distribution for the number 0.99 (see table of the standard normal distribution in [3] for example).

The control limits of the np-chart are given as the limit of the 99% random variation interval for the ratio of non-conforming parts by entering the estimations of the mean and standard deviation of the binomial distribution into these relations:

$$UCL = n \cdot \bar{p} + 2.58 \cdot \sqrt{n \cdot \bar{p} \cdot \left(1 - \frac{n \cdot \bar{p}}{n}\right)}$$

$$LCL = n \cdot \bar{p} - 2.58 \cdot \sqrt{n \cdot \bar{p} \cdot \left(1 - \frac{n \cdot \bar{p}}{n}\right)}$$

Here, $n \cdot \bar{p}$ are the means of the non-conforming parts determined in pre-production, and n is the (constant) sample size.

EXAMPLE 5: Example of an np-chart

During the final inspection of injection valve production, valve units are regularly found with various non-conformities. One decides to monitor how these parts are processed using an np-chart.

Since the average number of non-conforming valve units is at first unknown, but estimated to be 5%, one chooses a constant sample size of 150 parts in order to fulfill the requirement for the minimum sample size $\left(n \geq \frac{400}{5} = 80\right)$.

The following numbers of non-conforming valve units are found in the first 10 samples:

Sample no.	1	2	3	4	5	6	7	8	9	10
Number of non-conforming parts	9	11	6	10	9	7	12	8	11	9

Here, only the number of non-conforming parts is of interested; a defective valve unit may display many defects.

The average number of non-conforming units is determined from the numbers identified:



$$n\bar{p} = \frac{9+11+6+\dots+9}{m} = \frac{92}{10} \approx 9.$$

The control limits of the np-chart for the sample of size can now be calculated using this value-
 $n=150$:

$$UCL = 9 + 2.58 \cdot \sqrt{9 \cdot \left(1 - \frac{9}{150}\right)} \approx 16.5$$

$$LCL = 9 - 2.58 \cdot \sqrt{9 \cdot \left(1 - \frac{9}{150}\right)} \approx 1.5.$$

The sample is considered to have exceeded the control limits if 17 or more non-conforming parts were found in one sample, or only one or no non-conforming parts were found. Here, falling below the lower control limit means that the performance of the entire process has considerably improved compared to the previous performance. One should try, in this case, to find out what caused this (man, machine, material, methods, milieu) and stabilize the improved conditions.

4.2 p-chart

The same basic ideas applied for the np-chart also apply to the p-chart. Instead of controlling the number of non-conforming units np, the p-chart controls the proportion p of non-conforming units.

One chooses an individual value p by dividing the number of non-conforming units $n \cdot p$ in a sample by the sample size n. It is beneficial to consider the proportion p instead of the number $n \cdot p$ of non-conforming units because one can now work with changing sample sizes n, if this is unavoidable for operational reasons.

The average proportion p of non-conforming units is given by dividing the sum of all non-conforming units from a sample by the sum of all the inspected units (sum of the sample sizes).

The control limits of the p-chart can be calculated using the control limits from the np-chart by dividing the latter by n.

$$UCL = \bar{p} + 2.58 \cdot \sqrt{\bar{p} \cdot \frac{1-\bar{p}}{n}}$$

$$LCL = \bar{p} - 2.58 \cdot \sqrt{\bar{p} \cdot \frac{1-\bar{p}}{n}}.$$

For a changing sample size, instead of the n the average sample size

$$\bar{n} = \frac{1}{m} \cdot \sum_{i=1}^m n_i \text{ is under the root sign:}$$

$$UCL = \bar{p} + 2.58 \cdot \sqrt{\bar{p} \cdot \frac{1-\bar{p}}{\bar{n}}}$$

$$LCL = \bar{p} - 2.58 \cdot \sqrt{\bar{p} \cdot \frac{1-\bar{p}}{\bar{n}}}.$$

\bar{n} = average sample size

Since the standard deviation of the binomial distribution depends on the sample size n, during SPC, it is recommended to recalculate the control limits if the sample size has



changed by more than 25%. In this case, insert the current sample size n into the formula instead of the average sample size \bar{n} .

Calculating the characteristics from pre-production data

The average proportion of non-conforming units in relation to the total number of units inspected:

$$\bar{p} = \frac{n_1 \cdot p_1 + n_2 \cdot p_2 + \dots + n_m \cdot p_m}{n_1 + n_2 + \dots + n_m} = \frac{\sum_{i=1}^m n_i \cdot p_i}{\sum_{i=1}^m n_i}$$

m = number of samples

n_1 = size of the first sample

n_2 = size of the second sample

$n_1 \cdot p_1$ = number of non-conforming units in the first sample

$n_i \cdot p_i$ = number of non-conforming units in the i -th sample

EXAMPLE 6: Example of a p -chart

We consider a machine that winds enameled copper wire onto spools, strips the wire ends and solders short pieces of strand wire onto these ends. The machine creates a certain proportion of non-conforming spools where, for example the winding protrudes out of the spool or individual strands jut out of a solder joint.

Since the machine conditions cannot be improved on short term, the machine is to be monitored using a p -chart in order to recognize at an early stage a rise in the proportion of non-conformities.

First, every tenth part produced was removed and inspected for non-conformities. The parts inspected during five shifts of a work week were considered the pre-production samples. The following table summarizes the results of the study.

Shift No.	Number of examined parts	Number of non-conforming parts	Number of non-conforming parts
1	116	8	6.9%
2	119	9	7.6%
3	108	7	6.5%
4	112	8	7.1%
5	120	8	6.7%
Sum	575	40	

The average proportion of non-conforming units is (see formula in 4.2)

$$\bar{p} = \frac{8 + 9 + 7 + 8 + 8}{116 + 119 + 108 + 112 + 120} = \frac{40}{575} = 0.07 = 7\%.$$



The average sample size is $\bar{n} = \frac{575}{5} = 115$.

Accordingly, the control limits per the p-chart are given by:

$$UCL = \bar{p} + 2.58 \cdot \sqrt{\bar{p} \cdot \frac{1-\bar{p}}{\bar{n}}} = 0.07 + 2.58 \cdot \sqrt{0.07 \cdot \frac{1-0.07}{115}} \approx 0.13 = 13\%$$

$$LCL = \bar{p} - 2.58 \cdot \sqrt{\bar{p} \cdot \frac{1-\bar{p}}{\bar{n}}} = 0.07 - 2.58 \cdot \sqrt{0.07 \cdot \frac{1-0.07}{115}} \approx 0.01 = 1\%.$$

The control limits are entered onto the control chart as solid lines. The series of points on the control chart show that the machine is running in a very stable manner with the proportion of non-conforming parts determined (see Figure 4.1).

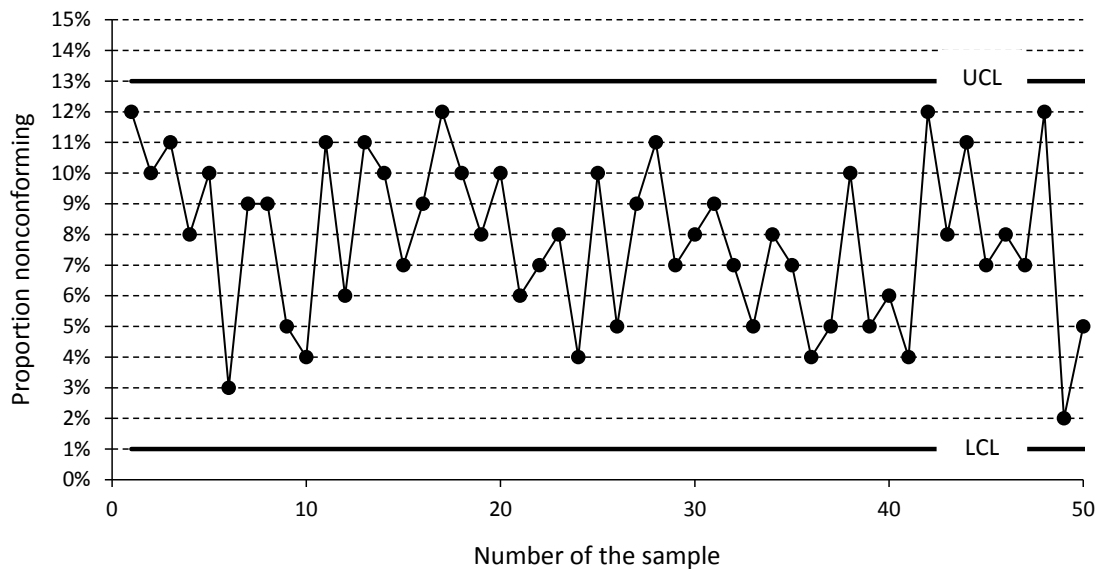


Fig. 4.1: Example of a p-chart ($\bar{n} = 115$, $\bar{p} = 7\%$)

4.3 c-chart

The c- and the u-chart (see Chapter 4.4) form, to an extent, a separate group among the control charts for discrete characteristics.

As opposed to the np- and p-charts, what is of interest here is not the number or proportion of non-conforming units, but rather the number c or the proportion u of non-conformities per product unit.

Examples for this are:

- Number of faulty soldering joints per circuit board.
- Number of assembly defects per assembly group.
- Number of metal shavings per cm² of test filter surface.

It is possible to combine the products of a sample into an inspection quantity and to monitor the number (or the proportion) of non-conformities in this inspection quantity.

We will first consider the c-chart.



The number c of non-conformities per sample is generally described by a Poisson distribution.

Its probability function f indicates as to which probability c non-conformities will be found in the sample, if the average number of errors is μ :

$$f(c; \mu) = \frac{\mu^c \cdot e^{-\mu}}{c!}.$$

The mean of the Poisson distribution is μ , its standard deviation is $\sigma = \sqrt{\mu}$.

If one collects - as described above - a sufficient number of results of products into an inspection quantity (sample), so that on the average four or more non-conformities are found in this inspection quantity (compare with minimum sample size), the normal distribution formally approximates the Poisson distribution very well. Similar to the concepts in Chapter 4.2 concerning the np-chart, one obtains the control limits for the c-chart from the 99% random variation interval of the number c of errors per sample, by inserting

$$UCL = \hat{\mu} + 2.58 \cdot \hat{\sigma}$$

$$LCL = \hat{\mu} - 2.58 \cdot \hat{\sigma}$$

the estimations $\hat{\mu} = \bar{c}$ and $\hat{\sigma} = \sqrt{\mu} = \sqrt{\bar{c}}$ for the mean and the standard deviation of the Poisson distribution into the relationships:

$$UCL = \bar{c} + 2.58 \cdot \sqrt{\bar{c}}$$

$$LCL = \bar{c} - 2.58 \cdot \sqrt{\bar{c}}.$$

Determine the average number of errors \bar{c} in the samples with the following formula:

$$\bar{c} = \frac{c_1 + c_2 + \dots + c_m}{m} = \frac{1}{m} \cdot \sum_{i=1}^m c_i.$$

m = number of samples

c_1 = number of non-conformities in the first sample

c_i = number of non-conformities in the i -th sample

EXAMPLE 7: Example of a c-chart

The rinsing fluid used to clean pump housings is filtered in order to remove metal shavings. A portion of the surface of a testing filter is examined under a microscope in order to find out if the amount of shavings exceed a given amount.

The number of shavings observed per cm² of filter surface is a quality characteristic and is to be monitored using a c-chart.

The examination of the last eight filters had the following results:

Filter no.	1	2	3	4	5	6	7	8
Number of shavings	11	13	8	12	11	9	14	10



On average, $\bar{c} = \frac{11+13+8+\dots+10}{8} = \frac{88}{8} = 11$ shavings were found.

The control limits for the provided c-chart are then:

$$UCL = 11 + 2.58 \cdot \sqrt{11} = 19.6$$

$$LCL = 11 - 2.58 \cdot \sqrt{11} = 2.4.$$

A "control" can be implemented if 20 or more, or two or less, shavings have been found.

4.4 u-chart

If one divides the number c of non-conformities in an inspection quantity (sample) by the number of units inspected in this sample, one obtains the proportion of non-conformities u .

The average non-conformities per unit \bar{u} is determined by dividing the average number of non-conformities in a series of m samples $\bar{c} = \frac{1}{m} \cdot \sum_{i=1}^m c_i$ by the average sample size

$$\bar{n} = \frac{1}{m} \cdot \sum_{i=1}^m n_i :$$

$$\bar{u} = \frac{\bar{c}}{\bar{n}} = \frac{\frac{1}{m} \cdot (c_1 + c_2 + \dots + c_m)}{\frac{1}{m} \cdot (n_1 + n_2 + \dots + n_m)} = \frac{c_1 + c_2 + \dots + c_m}{n_1 + n_2 + \dots + n_m} = \frac{\sum_{i=1}^m c_i}{\sum_{i=1}^m n_i}.$$

m = number of samples

n_1 = size of the first sample

n_i = size of the i -th sample

c_1 = number of non-conformities in the first sample

c_i = number of non-conformities in the i -th sample

The calculation of the average proportion of non-conformities \bar{u} with regard to the total number of units inspected occurs based on the pre-production data.

Like the c-chart, the control limits of the u-chart are based on an approximation of the distribution of u by the normal distribution.

However, they cannot be calculated directly from the control limits of the c-chart. They are derived using mathematical calculation rules for means and variances.

In contrast to the c-chart, the u-chart is beneficial because varying sample sizes are permitted.

Calculating the control limits:

$$UCL = \bar{u} + 2.58 \cdot \sqrt{\frac{\bar{u}}{\bar{n}}}$$

$$LCL = \bar{u} - 2.58 \cdot \sqrt{\frac{\bar{u}}{\bar{n}}}$$

\bar{n} = average sample size



The standard deviation depends on the sample size in this case. The control limits are to be recalculated if the sample size changes by more than 25%.

Recalculating the control limits for differing sample size:

$$UCL = \bar{u} + 2.58 \cdot \sqrt{\frac{\bar{u}}{n}}$$

$$LCL = \bar{u} - 2.58 \cdot \sqrt{\frac{\bar{u}}{n}}$$

n = size of the current sample

EXAMPLE 8: Example of a u-chart

Occasional errors arise on a machine to print circuit boards in the form of snapped-off component connections or missing or damaged components, all of which are easy to identify during a visual inspection. Since a constant sample size is not possible in this case, the performance of the machine is to be monitored using a u-chart. Approximately 200 randomly selected circuit boards have been combined to create a sample (inspection quantity). The following numbers of non-conformities were detected on ten samples, produced and inspected within a week:

Sample no.	1	2	3	4	5	6	7	8	9	10
Sample size	180	200	240	210	170	190	230	200	220	200
Number of non-conformities	7	9	8	8	12	11	11	9	4	8

The average number of non-conformities is $\bar{u} = \frac{7+9+8+\dots+8}{180+200+240+\dots+200} = \frac{87}{2040} \approx 0.043 = 4.3\%$.

The average sample size is $\bar{n} = \frac{1}{10} \cdot (180 + 200 + 240 + \dots + 200) = 204$.

This results in the control limits:

$$UCL = 0.043 + 2.58 \cdot \sqrt{\frac{0.043}{204}} \approx 0.08 = 8\%$$

$$LCL = 0.043 - 2.58 \cdot \sqrt{\frac{0.043}{204}} \approx 0.005 = 0.5\%$$

4.5 Control characteristic curves of control charts for discrete characteristics

The meaning of control characteristic curves of control charts for discrete characteristics shall be explained using the p-chart example from Chapter 4.2.,

The use of a p-chart allow one to recognize a change in the average proportion of non-conforming units in a population (e.g. production quantity during one shift). It would certainly be extremely unpleasant if the control chart reacted to small deviations of the current proportion non-conforming p of the mean \bar{p} , since such small deviations can be produced quite naturally due to the randomness of the sample, and therefore also would frequently unnecessarily intrude on a process with a stable proportion non-conforming.

In this sense, an ideal p-chart that functions with 100% certainty should only react when limits are exceeded due to actual process changes, meaning, for example $p' > 10\%$.



Otherwise, the chart should not react with the same certainty. A real p-chart does not possess these properties. Even an unchanged proportion non-conforming p' has a 1% probability that the p-chart reacted incorrectly (type 1 error).

Real changes in the proportion non-conforming are only displayed with a certain probability that becomes greater the more the actual value p' differs from the mean \bar{p} . This control probability related to the actual proportion non-conforming p' is described by a control characteristic curve (also called an operating characteristic).

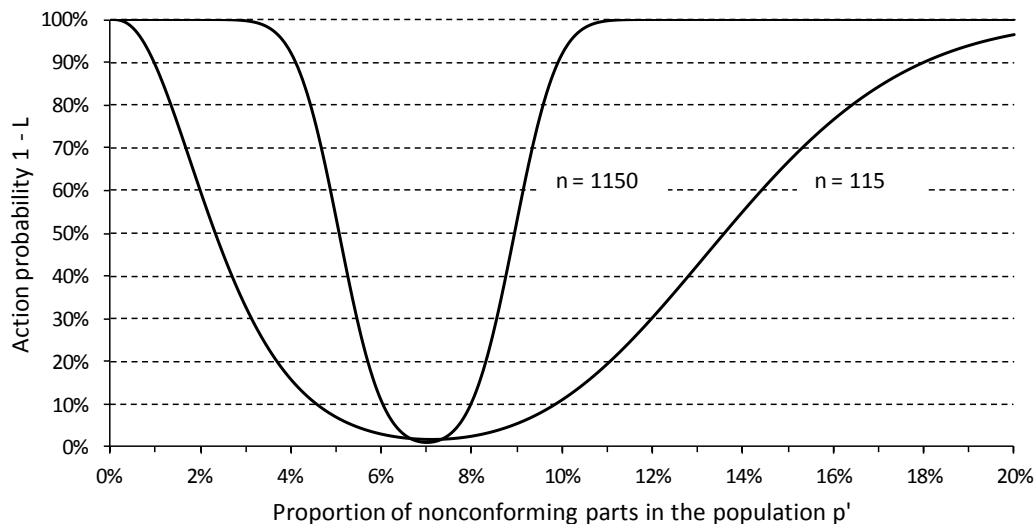


Fig. 4.2: Control characteristic curves of a p-chart for two sample sizes

From Figure 4.2, it is obvious that this probability of intervention increases as the proportion non-conforming increases (above 7%), but it only approaches the value 100% with a comparably large value of p' . At $p' = 13.5\%$, it is about 50%, meaning that in only half of all samples will an increase to $p' = 13.5\%$ in the proportion non-conforming lead to the curve exceeding the upper control limit.

The control characteristic curve depends on the chosen sample size n . Using the control limit formulas allows one to understand that the distance between the lower and upper control limits decreases with increased sample size n . The control characteristic curve becomes steeper near \bar{p} with growing n .

As an example, Figure 4.2 gives the control characteristic curve for the case $n = 1150$. For example, this sample size would result on average if 100% of the goods produced during a shift were inspected and considered the “sample”. A sample size that is too large obviously leads to the chart being more sensitive than otherwise desired.

This is why the average number of non-conforming parts in the sample should not be larger than 100 in control charts for discrete characteristics. A sample size of $n \leq \frac{10000}{\bar{p}}$ (with \bar{p} in percent) is created for the p-chart, accordingly.

Note that in this context, it is not unusual to inspect 100% of the production of a shift or day and to observe the proportion of non-conformities over a longer period of time. It is, however, not sensible to use control limits calculated statistically, since the population was completely inspected and the actual number of non-conformities is known. Instead, one might determine, at one’s own discretion based on experience with the process, a fraction non-conforming of defects that is not to be exceeded in this case. The previous



considerations can be transferred to the remaining control charts for discrete characteristics.

In the literature, occasional comparisons are made between control characteristics of control charts for discrete characteristics and those for continuous characteristics. Within Bosch, it has been determined (see Book 7) that control charts for continuous characteristics are to be used for characteristics that are measurable and that should be regulated (monitored) using a quality control chart. Therefore, discussions about the advantages and disadvantages of the control performance (operating characteristic curve) of control cards for discrete characteristics is only of theoretical interest.

4.6 Non-conformity tally chart

The non-conformity tally chart is used when inspecting discrete characteristics if a unit (component, functional unit) is to be inspected with regard to several possible defect types.

A quality control chart for discrete characteristics is provided as a form for this purpose. To prepare the chart, the types of defects are listed in the first column of the chart and the necessary details are entered into the header of the chart.

The numbers of defects of each type determined in the individual, consecutive inspections are entered in the following columns numbered 1 to 25. The inspections may include a sample or examine 100% of the parts. In the first case, the numbers 1 ... 25 correspond to one sample each, in the second case they correspond to the entire amount produced during a shift.

The sample size should be chosen sufficiently large for a selective inspection that there is a large enough probability, according to previous experience with the process at hand, of even finding defective parts or defects in the sample. A non-conformity tally chart where the number zero is almost the only occurrence is either proof of a very good production process or the result of a too small sample size.

The total number of the defects or non-conforming parts found in a sample is calculated by summing up the values within the columns and recording them in the corresponding row. Provided that the sample size (or the number of parts produced per shift) changes from test to test, it is sensible to also calculate the proportion non-conforming or non-conforming parts in the sample by dividing the number of non-conforming units by the sample size or the number of inspected units.

The number (proportion) of non-conforming units/defects can be represented graphically in the diagram provided on the chart. The behavior of the quality level is easy to see in this way. Since this representation is related to the sum of defects of different types, i.e. each type of defect is taken into equal consideration, it is hardly sensible to calculate statistical control limits at this point. If need be, a plausible (based on experience) upper limit can be inserted or drawn that is not to be exceeded under any circumstance.

To evaluate the chart, the numbers of defects of a type of defect are summed up and entered into the second to last column from the right. If one divides this sum of defects by the total number of all inspected units and multiplies this by 100. The result is a proportion non-conforming for each defect type in percent (average number of defects per inspected unit) that can be entered into the far right column (see Figure 4.3).

The last two columns show, which types of defects occur most often. This provides a place to begin when planning, which processes to improve. Note that the most commonly occurring defect does not necessarily cause the highest costs. Therefore, it might be sen-



sible to multiply the defect frequency with the scrap or rework costs per part and use this auxiliary quantity as a scale for improvements.


 BOSCH	Quality control chart for discrete characteristics					<input type="checkbox"/> x <input type="checkbox"/> np <input type="checkbox"/> c				Product/Part <i>EV4 Needle with armature</i> Process <i>100% Final Visual Inspection</i> Equipment <i>10x binocular</i>		Plant/Workshop <i>W600/625</i> Page no. 3.1	
Type of defect	1	2	3	4	5	6			25	Sum	%		
<i>Caulking burred</i>	1	4	6	5	-	1				30	0.03		
<i>Inlet hole dirty</i>	3	-	16	5	-	-				45	0.04		
<i>Armature damaged</i>	5	5	8	8	13	14				145	0.14		
<i>Armature lower edge burred</i>	5	2	8	14	19	5				85	0.08		
<i>Armature upper edge burred</i>	24	6	22	25	24	33				230	0.22		
<i>Needle collar burred</i>	-	-	-	53	1	5				247	0.24		
<i>4 or 5 cant burred</i>	136	34	164	125	75	108				1480	1.43		
<i>Guide damaged</i>	41	8	29	11	7	11				194	0.19		
<i>Armature spring guide bore burred</i>	7	25	18	14	23	12				244	0.24		
<i>Needle incomplete processed</i>	2	1	1	3	3	-				13	0.01		
<i>Needle dirty</i>	13	162	52	187	231	171				960	0.93		
Other defects	1	-	5	1	7	1				43	0.04		
Number of non-conformities	238	247	329	451	403	361				3716	3.59		
Number of non-conforming units													

Fig. 4.3: Schematic of a section from a quality control chart for discrete characteristics that is being used as a non-conformity tally chart



5 Acceptance sampling plans

Acceptance sampling plans are collections of short handling instructions, similar to statistical tests, to help to decide whether to accept or reject a lot. The handling instructions are therefore referred to as tests. A lot can, in this sense, be a quantity of goods delivered by a supplier or the goods produced during a shift, for example.

From the terms below, it is obvious that acceptance sampling plans were originally designed for use in the area of inspecting incoming goods.

A random sample is taken from the lot whose size depends on the total number of the units in the lot.

Significant characteristics of the units included the sample are then inspected with respect to their attributes, meaning each unit is, for example, assessed based on a gauge inspection (gap gauge, plug gauge) or by visual inspection based on boundary samples. Examples of the corresponding assessments are good/bad, OK/not OK, inside/outside the tolerance range.

If the number of units in the sample with a negative assessment is less than or equal to the acceptance number c specified in the sampling plan, then the entire lot will be accepted, otherwise it will be rejected. If a two-stage sampling plan is used, the result of the first sample will either dictate that the lot is accepted directly or that another sample is drawn and evaluated. The decision about the lot is then made using the results from both samples.

Within Bosch, such two-stage (multiple or consecutive) sampling plans are no longer used, since they fundamentally accept a certain proportion of non-conforming units in the lot. This does not conform with quality objectives for modern serial production (zero defect principle).

At Bosch, only a one-stage sampling plan with acceptance number $c=0$ is used, meaning only one sample is “taken” and the appearance of a non-conforming unit in the sample requires that the lot be rejected.

If the lot in question is a delivery of goods from a supplier, then the term “rejection” is self-explanatory. In the case of a selection inspection (sample inspection) in the production area using a sampling plan, rejection means that the quality of the inspected lot is insufficient. A negative sample results leads, in this case, to a 100% inspection.

We want to explain how to conduct an acceptance sampling plan in the following.

EXAMPLE 9:

The incoming goods department at a factory receives a delivery that includes $N=2000$ end shields for generators. The delivery includes 200 end shields, which were incorrectly provided with threads in one of the drill holes.

An employee in the QM department takes a random sample of $n=50$ parts and inspects their quality characteristics to see if they conform with the specifications. How great is the probability that 0, 1, 2, 3, ... non-conforming parts are included in the sample?

Since the sample is considerably smaller than the lot size, the interesting probabilities can be calculated using the probability function of the binomial distribution

$$P_{50i} = \binom{50}{i} \cdot 0.1^i \cdot (1 - 0.1)^{50-i}.$$



The number of non-conforming units in the lot is thus $p' = \frac{200}{2000} = 0.1 = 10\%$.

If one consecutively inserts the values 0, 1, 2, 3, ... for i in this formula, the following is produced

$$\text{for } i=0: \quad P_{50,0} = \binom{50}{0} \cdot 0.1^0 \cdot 0.9^{50} = 1 \cdot 1 \cdot 0.005 = 0.005 = 0.5\%$$

$$\text{for } i=1: \quad P_{50,1} = \binom{50}{1} \cdot 0.1^1 \cdot 0.9^{49} = 50 \cdot 0.1 \cdot 0.0057 = 0.029 = 2.9\%$$

$$\text{for } i=2: \quad P_{50,2} = \binom{50}{2} \cdot 0.1^2 \cdot 0.9^{48} = 1225 \cdot 0.01 \cdot 0.0064 = 0.078 = 7.8\%$$

$$\text{for } i=3: \quad P_{50,3} = \binom{50}{3} \cdot 0.1^3 \cdot 0.9^{47} = 19600 \cdot 0.001 \cdot 0.0071 = 0.139 = 13.9\%$$

$$\text{for } i=4: \quad P_{50,4} = \binom{50}{4} \cdot 0.1^4 \cdot 0.9^{46} = 230300 \cdot 10^{-4} \cdot 0.00786 = 0.181 = 18.1\%$$

$$\text{for } i=5: \quad P_{50,5} = \binom{50}{5} \cdot 0.1^5 \cdot 0.9^{45} = 2118760 \cdot 10^{-5} \cdot 0.873 = 0.185 = 18.5\%$$

This calculation should, of course, be completed for all further i up to $i=50$, however, this example should be enough to understand the process. See Figure 5.1 for a schematic with probabilities for the considered example. Their calculation is much simpler using the probability

function of the Poisson distribution $P_{n,i} = \frac{(n \cdot p')^i}{i!} \cdot e^{-n \cdot p'} = \frac{5^i}{i!} \cdot e^{-5}$.

The values obtained for both probability functions are compared in the following table.

i	$P_{n,i}$	
	Calculation with the probability function of the binomial distribution	Calculation with the probability function of the Poisson distribution
0	0.5%	0.7%
1	2.9%	3.4%
2	7.8%	8.4%
3	13.9%	14.0%
4	18.1%	17.5%
5	18.5%	17.5%



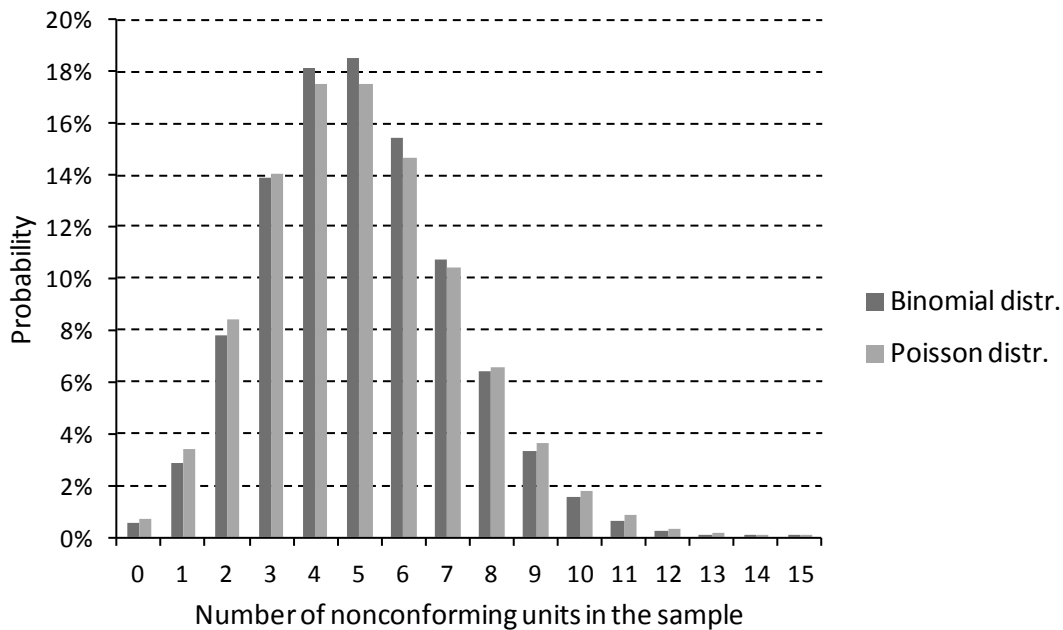


Fig. 5.1: Schematic of the probability function for Example 9

The question about probability L , with which at most three non-conforming parts are included in the sample, is easy to answer.

Since the cases

no non-conforming part in the sample

one non-conforming part in the sample

two non-conforming parts in the sample

three non-conforming parts in the sample

are mutually exclusive events, one merely has to add up the individual probabilities of the events for the cases $i=0,1,2,3$:

$$L = \sum_{i=0}^3 P_{50i} = P_{500} + P_{501} + P_{502} + P_{503} = 25.1\%.$$

5.1 Operating characteristics of acceptance sampling plans

The problem in Example 9 does not, of course, occur in reality, because the number of non-conforming parts contained in a delivery is unknown. In practice, a delivery is only accepted if no non-conforming parts are contained in the sample. The number of maximum accepted non-conforming parts in the sample is called the acceptance number c . In the Bosch single sampling plan $c=0$ is preset.

The previous considerations now allow one to calculate the probability L , with which a delivery with a proportion p' of non-conforming parts will be accepted. This is only the case if $i=0$ and thus $L=P_{50,0}$.

We use the probability function of the Poisson distribution with $i=0$ to calculate L :

$$L = P_{50,0} = \frac{(50 \cdot p')^0}{0!} \cdot e^{-50 \cdot p'} = e^{-50 \cdot p'}.$$



The values of L as a function of p' as represented graphically in Figure 5.2. The function L is called the operating characteristic. A graphical representation thereof is also called the operating characteristic curve or the acceptance characteristic curve.

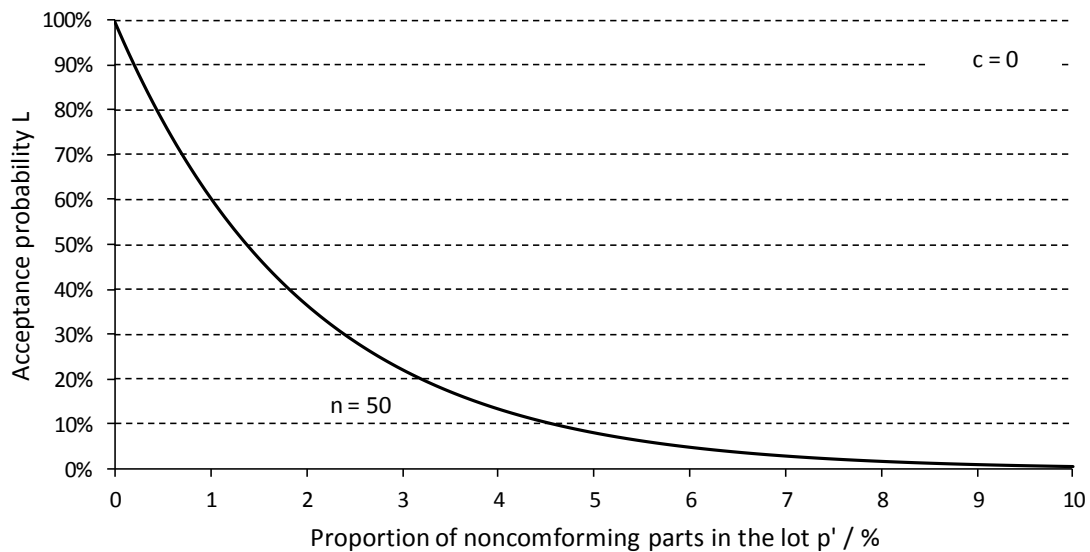


Fig. 5.2: Using the Poisson distribution to calculate the operating characteristic curve for $n=50$ and $c=0$

The acceptance probability for the example $p'=1.5\%$ in the figure is

$$L(n=50, p'=1.5\%, c=0) = e^{-50 \cdot 0.015} = 0.472 = 47.2\%.$$

A delivery with a proportion non-conforming of 1.5% will therefore be accepted with a probability of 47.2%. Or, said differently, on average about every second delivery with this proportion non-conforming will be accepted.

According to the Bosch single sampling plan, the sample size of a test depends on the size of the population N (order quantity). To be correct, one would always have to give L in the form $L(N, n, p', c)$.

For completeness, it should also be mentioned that there are also acceptance sampling plans with $c > 0$. In this case, to calculate the acceptance probability L , one must sum up the individual probabilities of the excluded cases $i=0, 1, 2, \dots, c$:

$$L = \sum_{i=0}^c P_{n,i} = P_{n,0} + P_{n,1} + P_{n,2} + \dots + P_{n,c}.$$

Depending on which of the requirements named in Chapter 3.3.4 has been fulfilled, the individual probabilities can be determined using the hypergeometric distribution, the binomial distribution or the Poisson distribution.



5.2 Characteristics of acceptance sampling plans

The operating characteristic of a test changes with the sample size n . With increasing n , the acceptance curve becomes steeper in the area of smaller proportions non-conforming, as Figure 5.3 shows for three sample sizes.

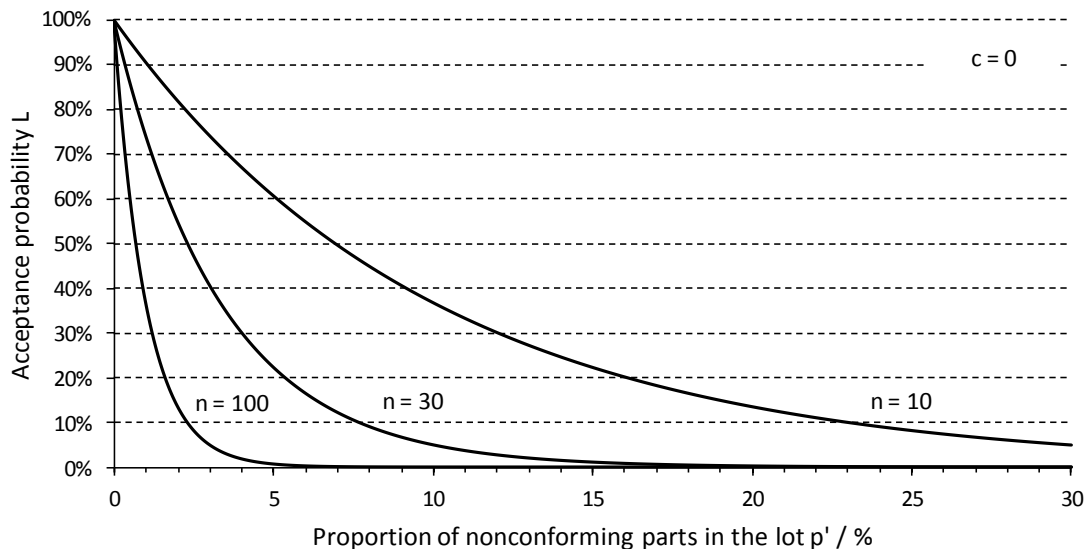


Fig. 5.3: Using the Poisson distribution to calculate operating characteristics for $c=0$ and various sample sizes n

An ideal sample plan with regard to a given limit would always reject lots whose true proportion of non-conforming units exceeded this limit.

A real sampling plan obviously does not fulfill this desire for ideal behavior in the sense of an on/off switch. Lots with a certain proportion non-conforming are not rejected with absolute certainty, rather they are rejected only with a certain degree of probability. While lots with an accepted proportion non-conforming are also only accepted with a certain probability.

The acceptance probability decreases more or less quickly from 100% to almost 0% with an increasing proportion non-conforming in the lot, depending on the sample size specified by the sampling scheme.

The proportion non-conforming p' that is accepted with a great probability $L(p'_A)$ of 90%, for example, is called the acceptable quality level (AQL). The complement to this acceptance probability $L(p'_A)$, thus the number $\alpha = 1 - L(p'_A)$, is called the supplier risk.

The proportion non-conforming accepted with only a small probability $L(p'_R)$ of 10%, for example, is called the limiting quality (LQ). $L(p'_R) = \beta$ is also called the buyer risk (see Figure 5.4).



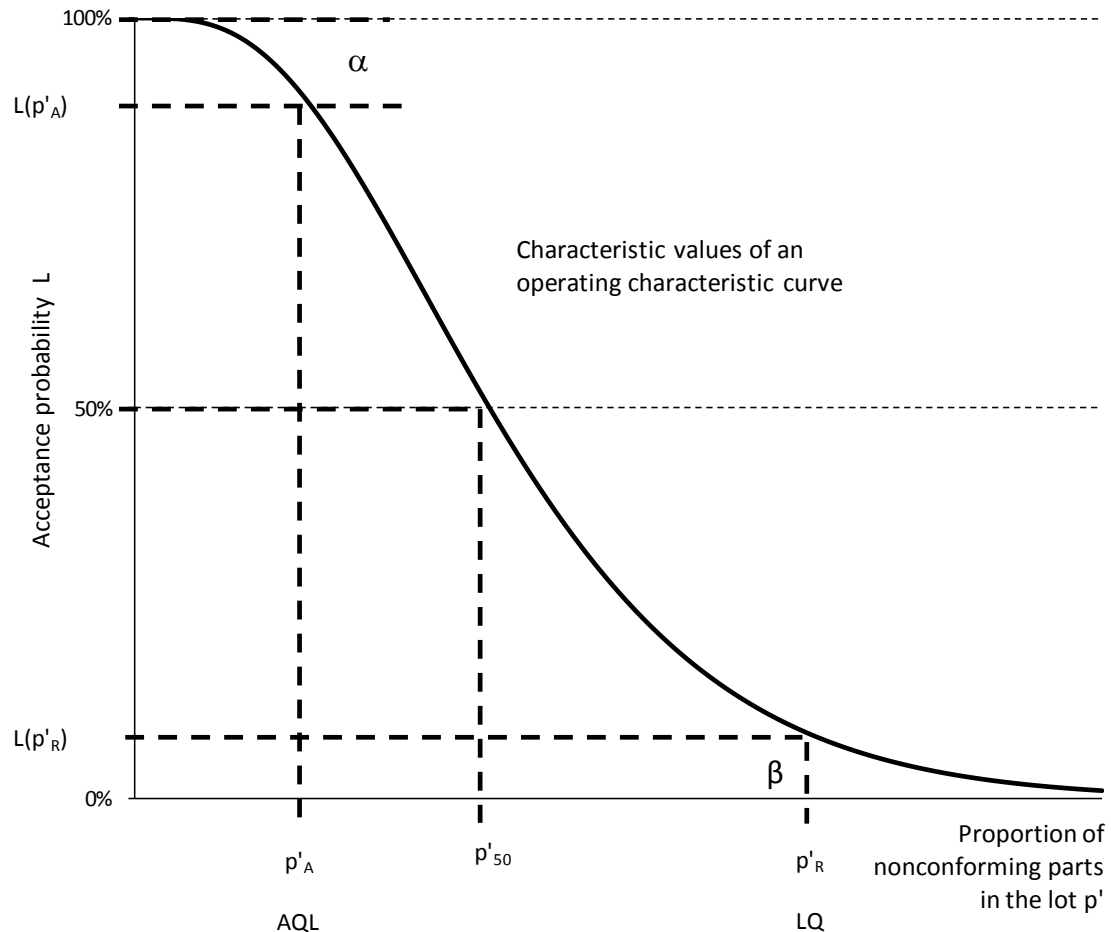


Fig. 5.4: Illustration of the characteristic values of an operating characteristic curve

The designation supplier risk is designed to express the idea that a certain risk exists for the supplier in that his delivery may be rejected by the buyer even though it only contains a relatively small proportion non-conforming. Correspondingly, the term buyer risk is intended to clarify that there is a certain chance of accepting a delivery although it contains a relatively large proportion non-conforming.

It should be emphasized that the AQL and LQ values of a test represent purely formal testing risks and are not to be misunderstood as a “permissible proportion non-conforming”. The calculated risk of a sampling inspection is independent of a legal claim of 100% conforming products in a delivery.

If one always inspects lots of the same size and the same quality p' according to the same sampling instructions (test with specified n and c), an average of $100 \cdot L\%$ lots will be accepted and $100 \cdot (1 - L)\%$ rejected. If the rejected lots are sorted to 100% and the non-conforming parts replaced with conforming ones, the total accepted quantity has a proportion non-conforming that is smaller than p' . In the long run, an average proportion non-conforming is accepted by the sampling inspection, the so called average outgoing quality (AOQ) D .

The function D is given by the product of the proportion of non-conforming units and the associated acceptance probability:

$$D = L(N, n, p', c) \cdot p'.$$



The average outgoing quality is zero when the delivered lots are conforming ($p'=0$). It also tends to zero with very high proportions non-conforming, since almost all of the lots will be rejected ($L \rightarrow 0$). D assumes a maximum D_{\max} with an average proportion non-conforming.

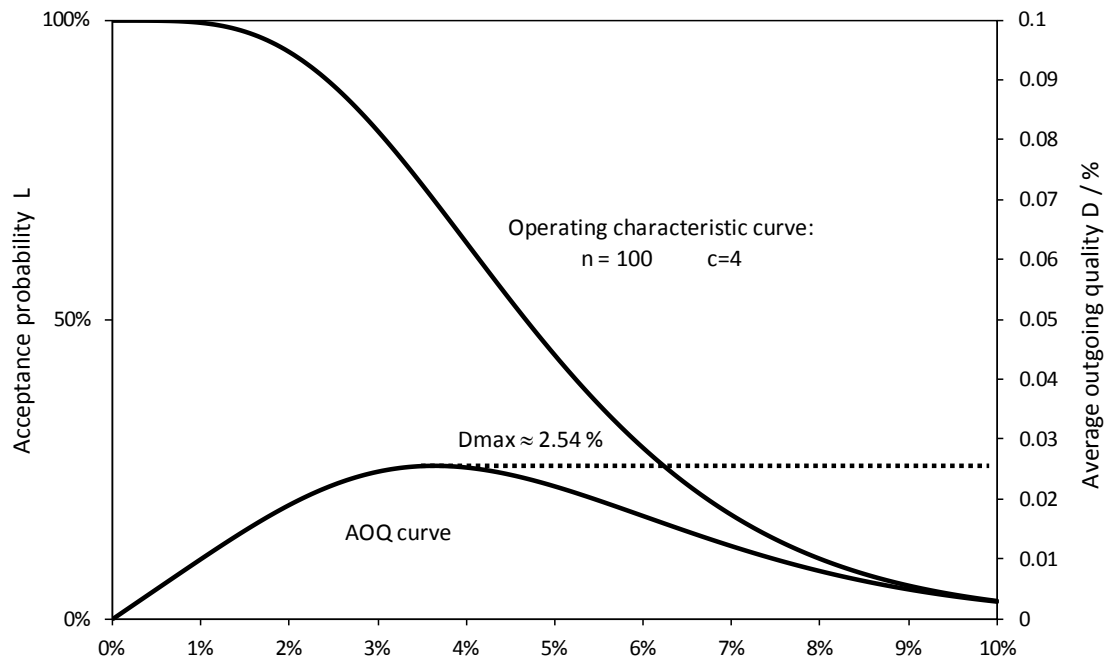


Fig. 5.5: Graphic representation of an operating characteristic curve and the associated average outgoing quality D . For illustrative purposes, a test with the acceptance number $c = 4$ was chosen.

D_{\max} can be easily calculated using the probability function of the Poisson distribution. By differentiating the function $D = L \cdot p' = e^{-n \cdot p'} \cdot p'$ and calculating the zero position of the derivative, it results that D has a maximum at the position $p' = \frac{1}{n}$. Entering this into the functional equation gives:

$$D_{\max} = \frac{1}{e} \cdot \frac{1}{n} \approx \frac{0.368}{n}.$$

EXAMPLE: From $N > 2000$ and $n = 60$ follows $D_{\max} \approx \frac{0.368}{60} = 0.61\%$.

The corresponding table value of the Bosch simple-sampling scheme is 0.6%. p'_{90} and p'_{10} can be calculated approximately by solving the equation $L = e^{-n \cdot p'}$ according to p' :

$$L = e^{-n \cdot p'} \Leftrightarrow \ln(L) = -n \cdot p' \Leftrightarrow p' = -\frac{\ln(L)}{n}.$$

If one enters $L = 0.1 = 10\%$ or $L = 0.9 = 90\%$ into the expression for p' , the result is:

$$p' = -\frac{\ln(0.1)}{60} = 0.038 = 3.8\% \quad \text{or} \quad p' = -\frac{\ln(0.9)}{60} = 0.0018 = 0.18\%.$$

The values agree sufficiently well with the corresponding table values. With smaller batch sizes, the approximation conditions found in Chapter 3.3.4 are no longer fulfilled. This means that using the Poisson distribution will yield large deviations from the table values.



5.3 Single sampling plan

As already stated at the beginning of Chapter 5, the single sampling plan is a collection of instructions (tests) that result in a lot's acceptance or rejection.

In contrast to double or multiple sampling plans, this decision is made using the results of just one sample within the single sampling plan.

Each test in the single sampling plan with acceptance number $c = 0$ is significantly characterized by the associated sample size. The quantities AQL, LQ and the maximum AOQ D_{\max} are given for every test in the single sampling plan. The AQL value there has the designation p'_{90} , the LQ value the designation p'_{10} . The indices 10 and 90 correspond to the associated acceptance probabilities in percent.

According to the single sampling plan, even large lot sizes (lot size $N \geq 5000$) can be worked on with the sample size n that is given in the far right column (calculated for $N = 5000$) for the respective test. The values given in the plan are calculated using the hypergeometric distribution.

A sample of n parts is taken from the lot. Provided that there are no non-conforming parts in the sample, the lot will be accepted (acceptance number $c = 0$).

A sample will have a high probability (90%) of not having a non-conforming part only if the proportion of non-conforming parts p' is very small. Hardly anything changes with the acceptance probability if one takes a sample of the same size from a larger lot that has considerably more parts, but the same proportion non-conforming p' .

This is due to the fact that accepting a uniformly mixed lot and taking a random sample will mean that each part has the same probability of being included in the sample (similar to drawing lottery numbers).

In practice, it is not suitable to take in lots if they originate from a production series with very different quality over time (e.g. periodically changing or constantly increasing or decreasing proportion non-conforming).


If this situation cannot be avoided, the lot to be inspected must be divided into subsets of $N = 5000$ parts maximum each, with each subset being subsequently sampled.

Make sure that the test specimens in a sample are taken from different places in the delivery. Each part must have the same chance of being taken (random selection). The values of p'_{90} , p'_{10} and D_{\max} help to decide, which test to use. Moreover, one must consider the average quality of the lot as well as the meaning of the testing characteristic(s) of, for example, function, quality and reliability.

Tests EOX, EOV and EOI are generally used as identification tests. They are only to be used as tests of quality for higher, more constant production reliability (e.g. inspecting a batch being hardened in a salt bath, punched parts, a selection test for surface treatment procedures).

Advances in series production with regard to the level of quality achieved are putting the use of sampling plans into question because the proportion non-conforming in ppm is extremely difficult to detect using acceptable sample sizes. Alternatively, one can use sampling plans for continuous characteristics (see [2]) to inspect discrete characteristics.



 BOSCH		Single sampling plan for inspection of discrete characteristics					
Order quantity N →		<100	101.....250	251.....500	501.....1000	1001.....2000	2001.....≥ 5000
Test EX	n	100%	100%	250	350	450	550
	p' ₉₀	-	-	0.03	0.02	0.02	0.018
	p' ₁₀	-	-	0.7	0.5	0.5	0.39
	D _{max}	-	-	0.1	0.1	0.1	0.06
Test EV	n	100%	100	120	160	200	250
	p' ₉₀	-	0.08	0.08	0.06	0.05	0.04
	p' ₁₀	-	1.8	1.7	1.3	1.1	0.9
	D _{max}	-	0.3	0.27	0.21	0.17	0.14
Test EII	n	45	55	70	90	110	130
	p' ₉₀	0.18	0.17	0.14	0.11	0.1	0.08
	p' ₁₀	3.8	3.6	3.0	2.4	2.0	1.7
	D _{max}	0.6	0.6	0.5	0.39	0.32	0.28
Test EI	n	20	24	30	40	50	60
	p' ₉₀	0.5	0.42	0.34	0.26	0.21	0.17
	p' ₁₀	9.8	8.7	7.2	5.5	4.4	3.7
	D _{max}	1.6	1.4	1.2	0.9	0.7	0.6
Test EOX	n	14	14	14	14	14	14
	p' ₉₀	0.7	0.7	0.7	0.7	0.7	0.7
	p' ₁₀	14	15	15	15	15	15
	D _{max}	2.4	2.5	2.5	2.5	2.5	2.5
Test EOY	n	6	6	6	6	6	6
	p' ₉₀	1.7	1.7	1.7	1.7	1.7	1.7
	p' ₁₀	31	32	32	32	32	32
	D _{max}	5.5	5.6	5.6	5.7	5.7	5.7
Test EOI	n	3	3	3	3	3	3
	p' ₉₀	3.4	3.4	3.4	3.4	3.4	3.5
	p' ₁₀	53	53	54	54	54	54
	D _{max}	10.4	10.5	10.5	10.5	10.5	10.5
Designations: n Sample size p' ₉₀ (AQL) non-conforming proportion (in %) in the lot with 90% acceptance probability p' ₁₀ (LQ) non-conforming proportion (in %) in the lot with 10% acceptance probability D _{max} (AOQL) max. AOQ (in %) i Number of non-conforming parts in the sample c Acceptance number, c = 0				Note: <ul style="list-style-type: none"> Only use EOX, EOY, EOI tests with high degree of production reliability! Take samples from various locations in the delivery! The part quantity applies for order subsets, goods produced during a shift are considered the order quantity in running production Examination result: Decision: i = 0 accept lot i > 0 reject lot			

Factory-internal sample sizes may be determined for special examinations, which differ from the specifications of this plan.



5.4 Acceptance characteristic curves for a single sample plan

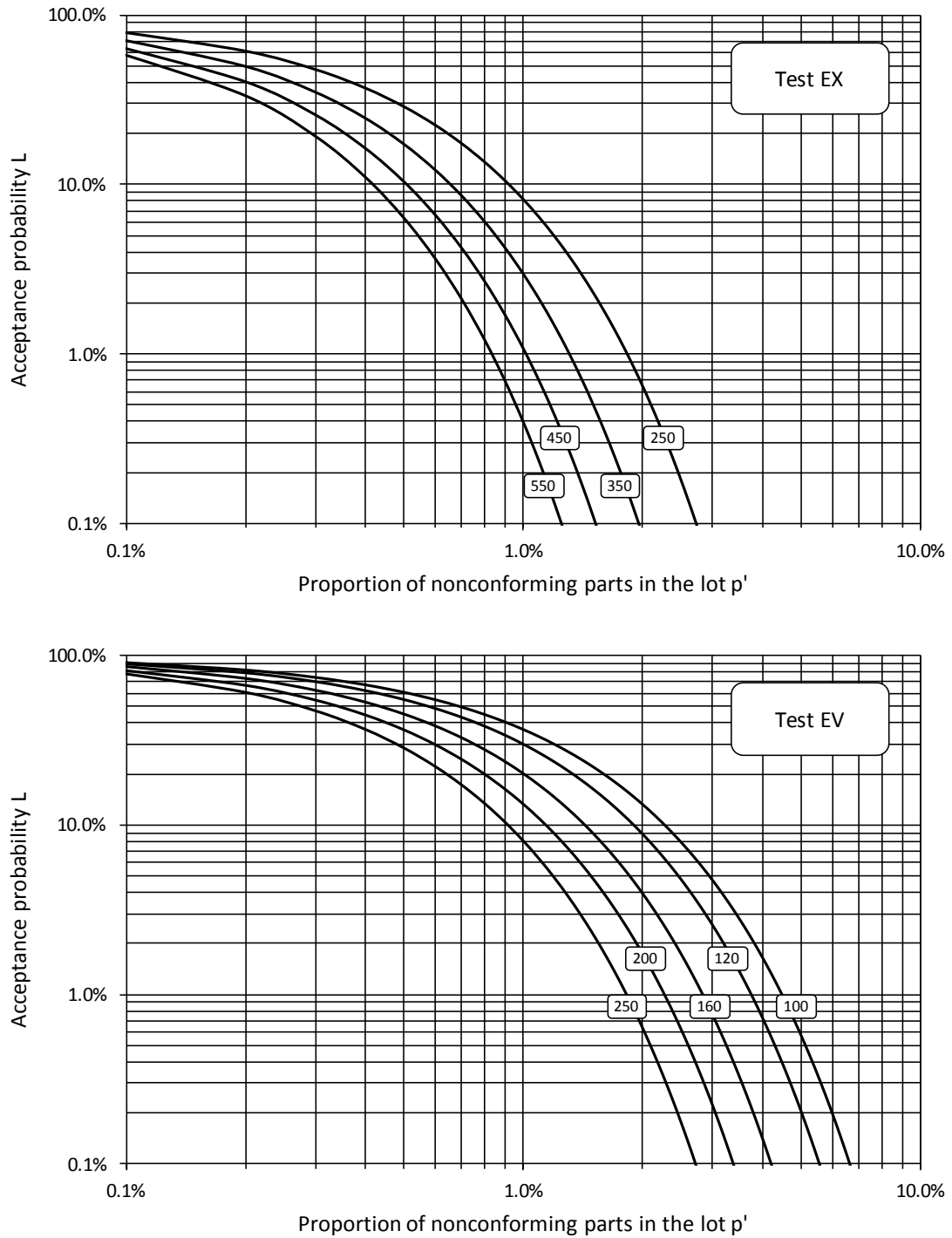
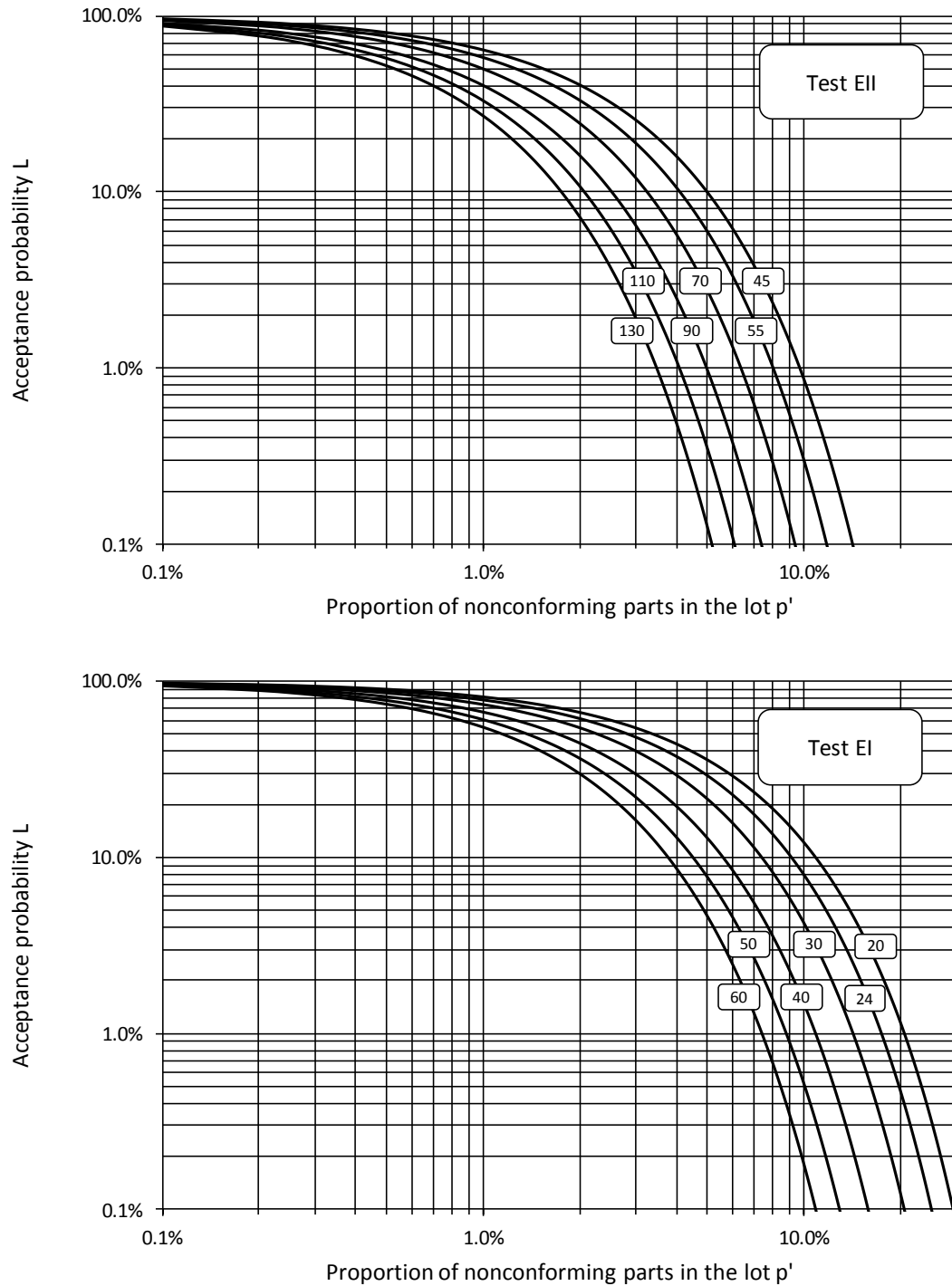


Fig. 5.6



**Fig. 5.7**

For the user interested in mathematics, we will provide some information about how to calculate the characteristics given in the single sampling plan.

Because of $c=0$, when calculating the acceptance probability L in the expression

$$L = \sum_{i=0}^c P_{n,i} = P_{n,0} + P_{n,1} + P_{n,2} + \dots + P_{n,c} \quad \text{only the characteristic } P_{n,0} \text{ is to be considered (see}$$

Chapter 5.1), from which simple expressions result when using the binomial or Poisson distributions.



Calculating using the binomial distribution:

$$L(c=0) = \binom{n}{0} \cdot p'^0 \cdot (1-p')^{n-0} = (1-p')^n \Leftrightarrow \sqrt[n]{L} = 1-p' \Leftrightarrow p' = 1 - \sqrt[n]{L}$$

$$\Rightarrow p'_{10} = 1 - \sqrt[n]{0.1} \quad \text{and} \quad p'_{90} = 1 - \sqrt[n]{0.9}$$

D_{\max} is given by calculating the zero position of the derivative of $D = L \cdot p'$ according to p'

and inserting the zero position into D : $D_{\max} = \left(\frac{n}{n+1} \right)^n \cdot \frac{1}{n+1}$.

Calculating using the Poisson distribution:

According to Chapter 5.2, $p' = -\frac{\ln(L)}{n}$ and $D_{\max} = \frac{1}{e} \cdot \frac{1}{n}$.

$$\Rightarrow p'_{10} = \frac{-\ln(0.1)}{n} \approx \frac{2.3}{n} \quad \text{and} \quad p'_{90} = \frac{-\ln(0.9)}{n} \approx \frac{0.105}{n} \quad D_{\max} = \frac{1}{e} \cdot \frac{1}{n} \approx \frac{0.368}{n}$$

Calculating use the hypergeometric distribution:

Calculating L and the characteristic values p'_{10} , p'_{90} and D_{\max} using the hypergeometric distribution is only possible through the use of a computer.

$$L(c=0) = P_{N,n,0} = \frac{\binom{N \cdot p'}{0} \cdot \binom{N \cdot (1-p')}{n-0}}{\binom{N}{n}} = \frac{\binom{N \cdot (1-p')}{n}}{\binom{N}{n}}$$

$$= \frac{[N \cdot (1-p')] \cdot [N \cdot (1-p') - 1] \cdot [N \cdot (1-p') - 2] \cdot \dots \cdot [N \cdot (1-p') - n + 1]}{N \cdot (N-1) \cdot (N-2) \cdot \dots \cdot (N-n+1)}$$

The first expression contains binomial coefficients in the general form $\binom{x}{n}$, whereby x

can be a non-integer depending on the values of N and p' and, even for small N , can therefore not be calculated using the factorial function on a calculator.

The numerator and denominator of the last expression consist of n factors each. Gradually calculating this expression of the form

$$L = \frac{N \cdot (1-p')}{N} \cdot \frac{N \cdot (1-p') - 1}{N-1} \cdot \frac{N \cdot (1-p') - 2}{N-2} \cdot \dots \cdot \frac{N \cdot (1-p') - n + 1}{N-n+1}$$

avoids the difficulty with the factorials as well as exceeding the allowed number range and an associated error message from the computer. Using a computer program, one can determine p'_{10} and p'_{90} by increasing p' , starting from zero and adding small increments until L assumes the value 0.9 or 0.1. D_{\max} is given by determining the respective maximum of the function $D = L \cdot p'$.

The acceptance characteristic in Figures 5.6 and 5.7 were calculated using the above formula for the binomial distribution. With small sample sizes, one can generate deviations using the table calculated using hypergeometric distribution.



6 Sampling inspection and 100% examination

A sampling inspection is the better the smaller the distance is between p'_A and p'_R . This corresponds to a growing steepness in the operating characteristic curve for small proportions non-conforming. However, with a fixed acceptance number $c=0$, this means an increase in the sample size n as well as examination costs. Advances in the area of series production have reached new levels of quality, a phenomenon that is putting the use of sampling plans in question, since proportions non-conforming in the ppm-range are hardly detectable anymore via acceptable sampling sizes.

In particular, it is often essential to test 100% of components that are related to safety. However, a 100% examination does not guarantee that the lot examined will be completely free of defects after the inspection. If people are conducting the inspection, as a rule, there will still be parts in the lot that are non-conforming because people become fatigued and lose focus. The relationship between the number of parts sorted out and the lot size is called “control efficiency”. It depends, among other things, on the proportion non-conforming originally in the batch (see Figure 6.1).

The increase in control efficiency with an increasing proportion non-conforming can be explained in that the examiner will remain more attentive if he finds a bad part “more often”. With very large proportions non-conforming, the control efficiency begins to degrade due to the appearance of fatigue. Figure 6.1 serves to present this fact qualitatively. The quantitative relationship is influenced, for example, by the working conditions, the motivation of the examiner and the degree of difficulty of differentiating between good and bad parts. It is also important, of course, how well the ratings “good” and “bad” of a quality characteristics are defined.

To this effect, instructions to “look for clean wiring” are certainly insufficient. Boundary samples are helpful in such cases.

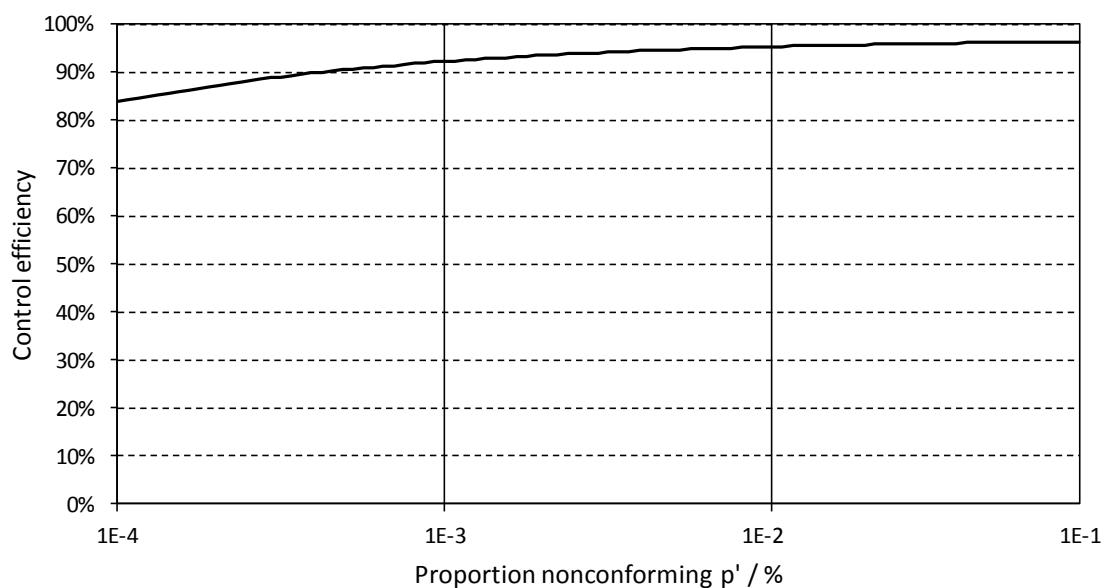


Fig. 6.1: Dependence of control efficiency on proportion non-conforming



Even double or repeated examinations of a lot do not offer any certainty that all of the non-conforming parts have been discovered. Since control efficiency also decreases with a decreasing proportion non-conforming, it becomes more unlikely to find another non-conforming part in the lot in subsequent examinations. From experience, even multiple examinations still result in a remainder of “bad” parts equal to about 0.2%.

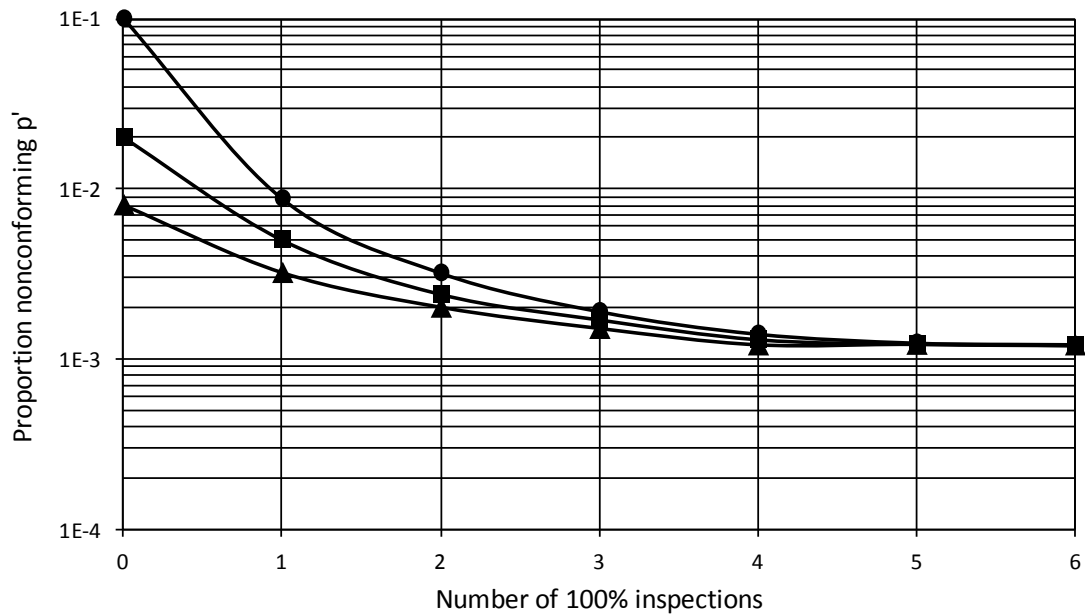


Fig. 6.2: Remaining ratio of non-conforming parts in the lot after repeated examinations. The initial values of the curve correspond to different original proportions non-conforming.



7 Confidence levels for the proportion non-conforming of the population

Based on Example 3 in Chapter 3.1, it was clear that the proportion non-conforming in a sample can differ from draw to draw.

The determined proportion non-conforming p in the sample is merely an estimation for the generally unknown proportion p' in the population. It is therefore not permissible to set p' equal to p .

It is, however, possible to generate a confidence level for p' . The confidence level is an interval that contains the quantity p' with a great probability.

The interval range (the “size” of the confidence level) depends on the proportion non-conforming p determined in the sample, on the size of the sample n and an arbitrary confidence coefficient P_A .

The confidence level is generally given in the form of an inequality:

$$p'_{lo} \leq p' \leq p'_{up}.$$

p'_{lo} is the lower limit, p'_{up} is the upper limit of the confidence level of p' . The probability that p' lies outside of the confidence level, meaning that it falls below the lower limit p'_{lo} or exceeds the upper limit p'_{up} , is called the significance level α . The following applies for this: $\alpha = 1 - P_A$.

Pearson and Clopper have calculated the confidence level of p' for various sample sizes and represented this in a diagram (see Figure 7.1). This diagram is only valid under the requirement $n \leq 0.1 \cdot N$. The significance level for each limit is 5%. Therefore, the confidence level in a two-sided case is

$P_A = 100\% - 2 \cdot 5\% = 90\%$ and $P_A = 100\% - 5\% = 95\%$ in a one-sided case.

The confidence level is also called the confidence range.

The lines drawn in the diagrams in Figure 7.1 are merely guides to facilitate reading.

With a sample of size $n=5$, the cases $i=0, 1, 2, 3, 4, 5$ of non-conforming parts in the sample will only yield the proportions $p=20\%, 40\%, \dots, 100\%$. Accordingly, only the points drawn at these percentages are relevant.



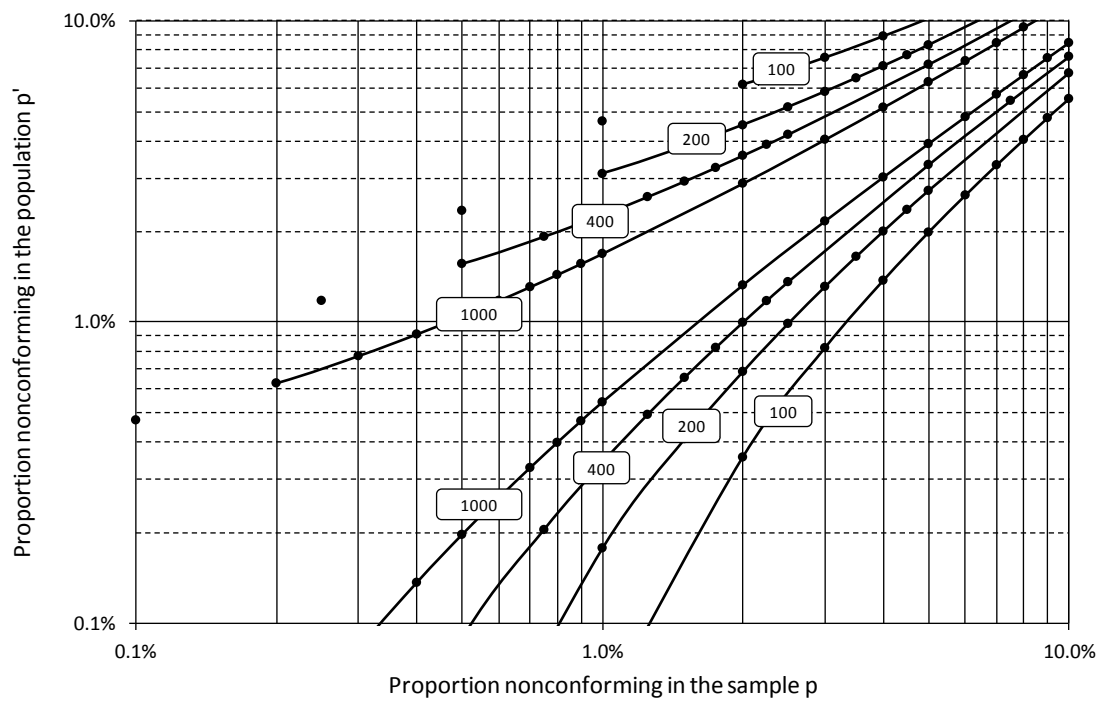
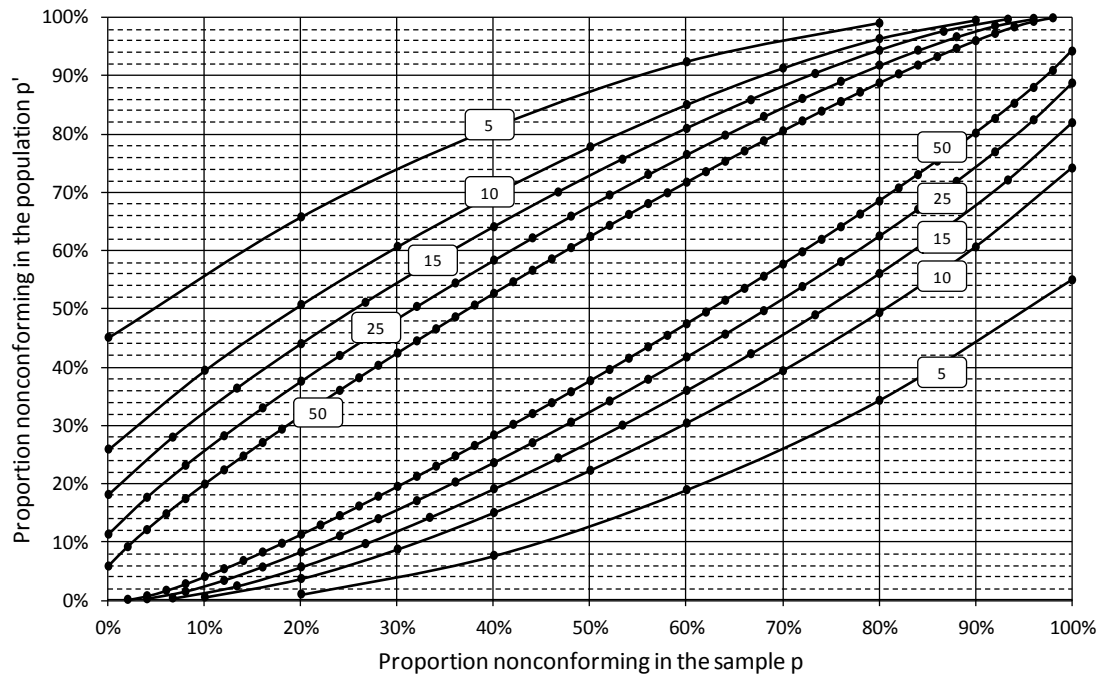


Fig. 7.1: Pearson-Clopper diagrams to determine the two-sided 90% confidence level for the ratio p' in the total population (with sample size n as a parameter).



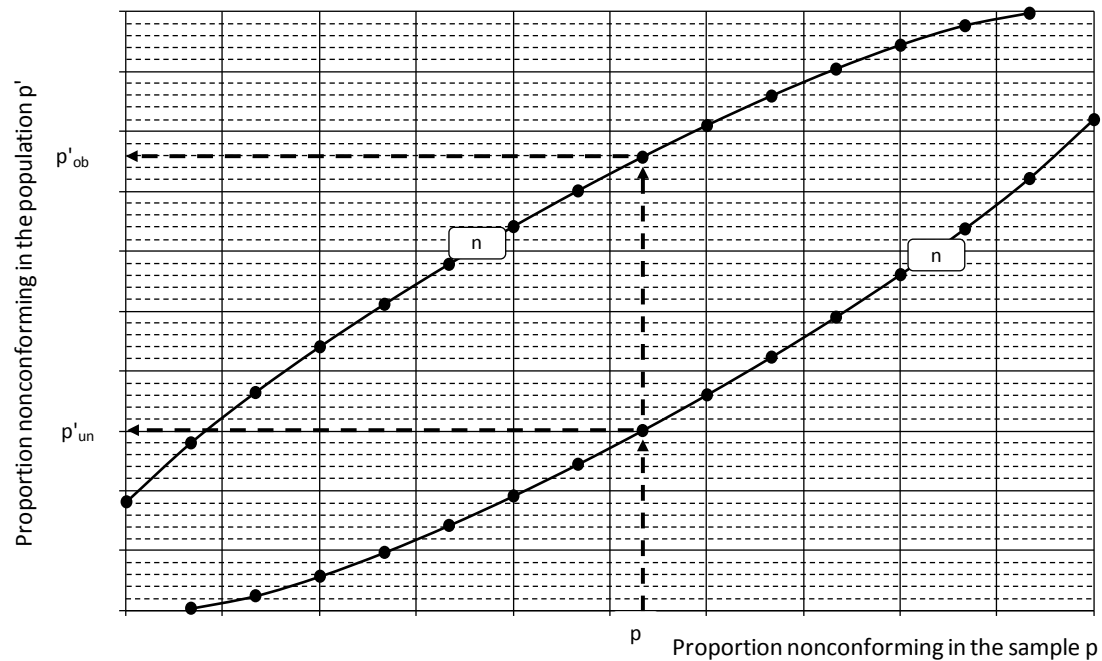


Fig. 7.2: Schematic for the use of the Pearson-Clopper diagram

Using the Pearson-Clopper diagram:

1. Determining the proportion non-conforming p in the sample ($p = \frac{i}{n}$)
2. Draw a vertical at point p on the p -axis
3. Determine the intersections with both “curves” at the sample size in consideration
4. Read the values p'_{lo} and p'_{up} on the p' -axis at the intersections

In general, the curves for the actual sample size of interest will not be contained in the diagrams. The diagrams from Figure 7.1 are, however, certainly sufficient for a rough estimate. Otherwise, refer to the calculation options or additional diagrams in the literature (e.g. [7]) on the following pages.

With the requirement of binomial distribution, one can calculate the limits of the two-sided confidence interval $p'_{lo} \leq p' \leq p'_{up}$ for the confidence level $1 - \alpha$ using the following formulas (see [7] and [9]):

$$p'_{lo} = \frac{i}{i + (n - i + 1) \cdot F_{f_1, f_2; 1 - \alpha/2}} \quad \text{with } f_1 = 2 \cdot (n - i + 1) \text{ and } f_2 = 2 \cdot i$$

$$p'_{up} = \frac{(i + 1) \cdot F_{f_1, f_2; 1 - \alpha/2}}{n - i + (i + 1) \cdot F_{f_1, f_2; 1 - \alpha/2}} \quad \text{with } f_1 = 2 \cdot (i + 1) \text{ and } f_2 = 2 \cdot (n - i).$$

If i is the number of non-conforming parts found in the sample of size n , then $p = \frac{i}{n}$. The given formulas apply to $1 \leq i \leq n - 1$. The following formulas apply to the special cases of $i = 0$ or $i = n$:

If $p = 0$ (i.e. $i = 0$) is $0 \leq p' \leq p'_{up}$ with $p'_{up} = 1 - \sqrt[n]{\alpha/2}$,

if $p = 1$ (i.e. $i = n$) is $p'_{lo} \leq p' \leq 1$ with $p'_{lo} = \sqrt[n]{\alpha/2}$.



EXAMPLE:

In a sample of size $n=50$, $i=13$ non-conforming parts were found. This corresponds to a proportion of $p = \frac{13}{50} = 26\%$. For the confidence level $1-\alpha=90\%$ (i.e. $\alpha/2=5\%$), with $f_1 = 2 \cdot (50-13+1) = 76$ and $f_2 = 2 \cdot 13 = 26$ as well as the associated table value of the F -distribution $F_{76,26;95\%} \approx 1.78$, yields the lower confidence limit

$$p'_{lo} = \frac{13}{13 + (50-13+1) \cdot 1.78} = 16.1\%$$

or with $f_1 = 2 \cdot (13+1) = 28$ and $f_2 = 2 \cdot (50-13) = 74$ as well as $F_{28,74;95\%} \approx 1.62$ the upper confidence limit

$$p'_{up} = \frac{(13+1) \cdot 1.62}{50-13+(13+1) \cdot 1.62} = 38.0\%.$$

If, in the case being considered ($n=50$), one does not find a non-conforming part in the sample ($i=0$), the following is valid for the true proportion p' of non-conforming parts in the population with a significance level of 5%:

$$0 \leq p' \leq p'_{up} \quad \text{with} \quad p'_{up} = 1 - \sqrt[50]{0.05} = 5.8\%.$$



8 Appendix

Symbols and terms

Table of uniformly distributed random numbers

References

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Symbols and terms

$\sqrt{\quad}$	Root sign
Σ	Summation symbol
\leq	Less than or equal to
\geq	Greater than or equal
\approx	Is approximately
c	Permitted number of non-conforming parts in the sampling plan with simple sampling inspection (acceptance number)
c_1	Acceptance number for the first sample
c_2	Acceptance number for the second sample
D	AOQ (Average Outgoing Quality)
D_{\max}	AOQL (Average Outgoing Quality Limit)
i	Number of non-conforming parts in a sample of size n
L	Probability of acceptance
$1 - L$	Probability of rejection
N	Lot size
n	Sample size
n_1	Size of the first sample
n_2	Size of the second sample
p'	Number of characteristic values in a lot of size N
p	Number of characteristic values in a sample
p'_A	Proportion non-conforming in the lot, in which the lot, with a probability of $A\%$ is accepted
p'_R $R\%$	Proportion non-conforming in the lot, in which the lot, with a probability of $R\%$ is rejected
p'_{50}	Indifferent quality level
P	Probability of an event
P_A	Confidence level
$q' = 1 - p'$	Proportion of good in the lot
$\alpha = 1 - P_A$	Significance level
$\alpha = 1 - L(p'_A)$	Supplier risk
$\beta = L(p'_R)$	Buyer risk
μ	Mean of a population
σ	Standard deviation of the population



Table of uniformly distributed random numbers

	Column									
Row	0-4	5-9	10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49
0	23052	48243	24624	15997	12116	98657	97569	71569	97290	06217
1	87660	05796	41005	30170	06495	33495	36969	83527	22258	66380
2	92056	11091	39175	29749	61767	86681	45480	32135	52686	36777
3	99114	61580	46603	25428	69350	40177	47267	70584	39605	89772
4	19829	30078	54474	89642	25107	65496	27020	34923	24120	39632
5	00257	56189	04388	43904	00269	03869	27470	46524	07060	79213
6	52884	91115	18989	21253	52430	06233	29212	35287	88464	74187
7	38823	01493	80692	23907	42125	33449	77961	94984	33215	81846
8	63225	11090	74120	01563	80747	36605	63691	06023	74849	92168
9	04856	12897	18850	60351	52041	44991	40379	02931	29009	15792
10	18155	21056	13422	92340	30365	28570	43923	83294	78145	41748
11	98242	87450	35716	38107	85891	26088	13332	24260	06642	61878
12	52673	14212	89681	95824	32194	89510	73434	65057	37310	92366
13	28771	56556	52158	82736	48909	85164	73906	75544	12981	40657
14	86104	91600	00744	18553	66034	10870	50746	25803	32331	49486
15	46530	00600	57985	62041	09421	37756	40128	79945	01821	25803
16	44378	19287	05116	06899	24108	99407	04730	11682	66873	32054
17	57572	72262	15102	99871	18359	81043	34552	79480	39861	25857
18	12305	11446	13644	70559	52831	62037	20086	50528	91558	36830
19	22510	94232	40393	85473	90794	81984	20128	46541	79145	39337
20	83111	29676	91863	57922	61584	52530	89171	97596	78692	28954
21	89910	90525	23878	08186	02464	70222	77207	54200	67798	39533
22	11594	28660	49170	04967	19161	51090	88064	34755	21566	20409
23	31659	44535	66765	34548	55678	96482	24594	14451	88479	89844
24	48423	52923	71139	73830	54291	08246	08637	20566	77117	39548
25	38946	77012	42168	20736	53853	05244	06620	26255	79011	04118
26	33914	74304	88057	36553	17315	69758	80703	62867	90952	64663
27	62479	36851	56090	83944	74260	06536	03710	85383	51126	74507
28	43857	08437	97253	76107	95003	81013	48361	06246	85868	97593
29	73314	98517	21423	67902	83504	70464	65161	19194	04953	54684
30	47305	84072	85154	86966	44738	23456	40383	77226	38579	47777
31	09927	02355	30915	85985	90229	31410	65750	98300	76094	86097
32	29384	41788	82679	85760	02636	58807	95330	19520	05006	47751
33	70021	60060	03703	76775	09843	75017	70571	52502	55758	29676
34	54750	37676	44117	70191	14446	53354	77381	00833	88671	46865
35	48548	87768	27483	75960	43253	01993	54940	75169	58513	16435
36	07509	69104	14445	30500	19959	67926	28194	34004	70850	48910
37	57840	92818	23163	46311	34046	79759	28326	34369	65828	18422
38	51381	28189	64074	90113	08031	73800	90213	73888	70004	84869
39	17224	39087	89394	98652	75330	15481	45766	99252	06301	44897
40	46921	41501	27624	95452	34771	68474	69458	02509	17047	71517
41	88102	66680	66351	84489	98890	33189	81778	08815	74502	97816
42	49233	73950	33481	61711	96374	33743	14554	86507	27965	57100
43	06157	98505	10789	29164	98387	48474	62190	37600	34005	05193
44	43059	38031	25733	72242	10579	85250	51987	45500	98119	85256
45	01206	48280	08241	75151	20140	82057	04259	08605	05865	49120
46	34217	60177	22855	05355	40398	97922	61407	05282	47333	82461
47	10247	99029	04580	18576	61429	67653	67768	72896	16318	31316
48	56087	93312	69915	78988	62339	13001	96733	09723	52424	41321
49	20334	71022	97069	62706	52937	13808	83818	25275	12313	94352



Table of uniformly distributed random numbers (continued)

	Column									
Row	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85-89	90-94	95-99
0	81063	48840	49585	81371	69254	71574	24286	00369	39178	62839
1	06868	61649	56507	24503	39761	92498	07999	23115	07205	25238
2	09911	36640	60022	29224	19215	40812	13340	29988	20609	86668
3	56366	51173	72451	77304	58466	81759	15976	21789	31661	75958
4	33144	89861	24828	29244	66531	25377	57700	73831	25509	83083
5	67524	74639	63887	46609	09893	88758	53505	14253	44827	70354
6	49768	95279	27107	46150	59060	87314	87682	06260	07681	23651
7	85748	91866	67644	14486	74300	67067	14210	01997	80698	10670
8	35068	48989	26700	40573	96764	05028	77579	70708	23955	31291
9	77948	97843	91396	11486	14606	93765	85726	01658	35412	68600
10	77522	24674	62214	42349	77011	38665	91845	13203	68666	33376
11	86964	58562	32668	70214	10042	42672	53845	25011	37367	84726
12	52896	12128	69356	32841	77106	84577	63687	24108	33057	30275
13	90869	27963	21534	43212	79036	65698	01354	19691	80080	27912
14	69922	01213	12562	50437	49102	89791	21765	65197	33086	65396
15	60660	99942	00060	87017	63407	70815	43045	74454	80056	18844
16	65259	48911	61274	01822	60161	93475	23829	59465	47991	65667
17	45505	99205	47287	27764	99397	51848	18446	70538	54561	20858
18	23610	66464	46231	85481	07976	75277	56745	53838	04772	06283
19	28162	08150	73322	45027	76538	43435	23671	66372	36829	08832
20	67018	46682	82628	50507	86056	20875	59825	38760	56695	90245
21	25028	01378	21068	01895	94855	93510	01408	76503	84427	51950
22	72248	86042	01018	17327	81929	44773	47022	56794	18463	78029
23	07869	67993	20519	21254	97380	59230	76180	11895	38977	46836
24	95283	33355	63965	80130	22010	57971	60440	72815	25354	50157
25	83996	45206	87192	49381	96561	45002	36514	79262	40605	04549
26	63396	61623	33875	46356	12583	96858	02275	84846	00810	74554
27	82737	33969	39743	86170	08214	32621	96890	81702	37739	60019
28	62532	60921	25055	79189	40988	01989	42503	20000	00344	00246
29	92938	00021	62631	77949	20404	53889	31776	87542	94112	11284
30	92163	01840	24296	95823	42477	71399	24102	34082	33144	85359
31	82052	58508	07576	27130	04037	22681	41782	68776	23184	48763
32	56269	01164	01249	35781	49097	63223	10951	32640	18353	30281
33	05444	83487	02665	69588	13857	45096	27022	07246	34935	87710
34	34246	56529	63892	82535	08560	39909	01687	13356	78215	58027
35	33788	08985	88608	38279	52472	08687	32878	11956	42187	65732
36	50513	91800	75364	88296	25549	94138	87102	14889	71115	45573
37	22348	52082	66569	25934	96691	04926	66624	60681	28640	25272
38	84865	22743	80597	11959	01114	89071	73403	98546	80052	90712
39	85411	87416	40597	21356	91908	57225	52952	12140	10643	75146
40	37413	31794	32382	38200	79472	03939	70620	91482	06762	70991
41	27019	20297	02471	22637	08455	37761	59291	98680	11263	23649
42	02694	70423	52261	74799	31107	49616	20747	51509	41116	18250
43	87314	95138	25460	32941	09847	03802	36890	41406	39428	77854
44	22984	74790	76474	84295	92192	03710	98500	78493	87191	22803
45	18650	72743	71909	66768	21687	23253	88932	55826	09244	33864
46	73911	43490	25188	23548	34900	92446	09181	67548	66539	97764
47	20690	41250	20542	61036	50562	45634	55762	28606	92830	52986
48	39595	01975	13008	02100	55605	20239	04690	66195	70139	28889
49	85944	45469	33030	44230	27312	65585	06685	95747	19440	31179



References

- Bosch series “Technical Statistics” (C/QMM)
- [1] Book 1: Basic Principles of Technical Statistics, Continuous Characteristics
 - [2] Book 3: Evaluating Measurement Series
 - [3] Book 4: Statistical Procedures, Forms and Tables
 - [4] Book 7: Statistical Process Control (SPC)
 - [5] Brochure: Elementary Quality Assurance Tools (C/QMM)
 - [6] M. Sadowy: Industrielle Statistik, Vogel-Verlag, Würzburg, 1970
 - [7] Graf, Henning, Stange, Wilrich: Formeln und Tabellen der angewandten mathematischen Statistik, Springer-Verlag, Berlin, 1987
 - [8] Lothar Sachs: Angewandte Statistik, Springer-Verlag, Berlin, 1992
 - [9] Hartung: Statistik, Oldenbourg, München, 1989
 - [10] Erwin Kreyszig: Statistische Methoden und ihre Anwendungen, Vandenhoeck u. Ruprecht, Göttingen, 1988
 - [11] H. Weber: Einführung in die Wahrscheinlichkeitsrechnung und Statistik für Ingenieure, Teubner, Stuttgart, 1988
 - [12] Karl Bosch: Elementare Einführung in die Wahrscheinlichkeitsrechnung, Vieweg, Braunschweig, 1989
 - [13] Karl Bosch: Elementare Einführung in die angewandte Statistik, Vieweg, Braunschweig, 1989
 - [14] W. Uhlmann: Statistische Qualitätskontrolle, Teubner, Stuttgart, 1982
- Deutsche Gesellschaft für Qualität e.V. (Hrsg.):
- [15] DGQ-Schrift 11-04, Begriffe im Bereich der Qualitätssicherung
 - [16] DGQ-Schrift 16-01, Stichprobenprüfung anhand qualitativer Merkmale
 - [17] DGQ-Schrift 16-26, Methoden zur Ermittlung geeigneter AQL-Werte
 - [18] DGQ-SAQ-Schrift 16-37, Stichprobenprüfung für kontinuierliche Fertigung anhand qualitativer Merkmale
 - [19] DGQ-Schrift 18-18, Anleitung zur Qualitätsregelkarte und zur Fehlersammelkarte
 - [20] DGQ-Schrift 16-30, Qualitätsregelkarten
 - [21] DGQ-Schrift 16-31, SPC1 - Statistische Prozeßlenkung
 - [22] DGQ-Schrift 16-32, SPC2 - Qualitätsregelkartentechnik
 - [23] DGQ-Schrift 16-33, SPC3 - Anleitung zur Statistischen Prozeßlenkung (SPC), Beuth-Verlag, Berlin



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