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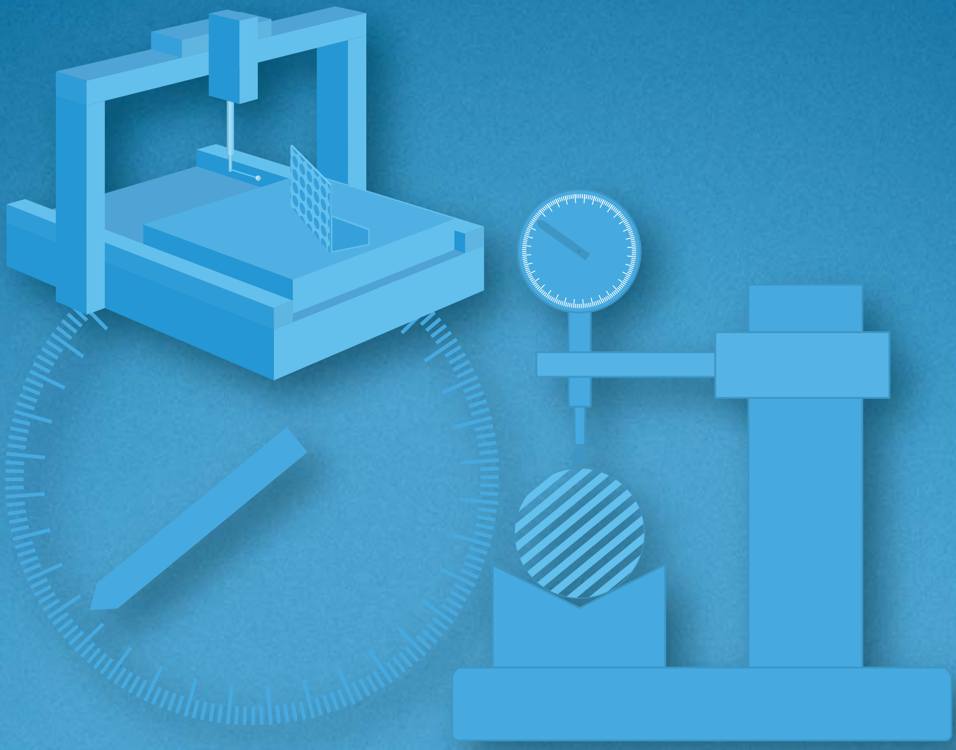

Quality Management in the Bosch Group | Technical Statistics

8. Measurement Uncertainty

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Quality Management in the Bosch Group
Technical statistics

Booklet 8 – Measurement Uncertainty

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1 Introduction

An uncertainty must always be specified for every measurement result. This is a requirement which is deduced from the standards [ISO 9000], [ISO 10012], [ISO 14253], [ISO 17025] and [DIN 1319-1] among others. The application for which the measuring device is being used and with which a measurement result is determined is irrelevant. In particular, it is essential to have knowledge of and to state the measurement uncertainty in any qualified decision that is made on the basis of measurement results.

The term "measurement uncertainty" is defined in the "International Vocabulary of Metrology" as a *"Non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used"* [VIM, 2.26]. The shorter term "uncertainty" is also used in place of the term "measurement uncertainty" in the literature.

The terms used in this issue have been taken from [VIM], [ISO 3534-2], [ISO 3534-1], [ISO 9000], [ISO 14253], [GUM], [DIN 1319-1] and [DIN 1319-4]. The chapter *Definition of terms* contains a compilation of the most important standardized definitions.

The possibilities for determining measurement uncertainty are varied and can therefore not be represented in a generally applicable algorithm. Thus, this booklet is divided into the chapters 1 to 6 with essential minimum information for each user, and the appendix. Some examples for the calculation of measurement uncertainty are included in the appendix. Relevant literature should be referenced for many more examples.

This booklet is primarily based on the "Guide to the expression of uncertainty in measurement" [GUM] [1]. In contrast to the previous edition of this booklet, conformity to [GUM] is established consistently and the specification of a model equation is required as a basic principal. Among other things, this ensures a clear and systematic approach. The approaches denoted as "simplified procedures" in the previous version are presented in an appropriately adapted way without the mathematical work having been increased (cf. chapters 4.3.1 and 4.5). In addition, requirements which are often more stringent, particularly in the fields of development, have been taken into consideration and also these more complex procedures are presented in greater detail. However, the explanation of how to determine measurement uncertainty in case of interdependent (correlated) measurands has largely been disregarded because of the increased mathematical workload involved. The appendix describes only the basic fundamentals and the calculation algorithm.

The procedures described here do not provide parameters for the distribution of the individual measured values of a measurand. Instead, they provide an estimate of the range of values within which the true value of the measurand associated with the individual measured values is expected with a certain confidence level, however, without knowing this true value exactly. This initially appears to contain a contradiction of the definition of measurement uncertainty according to [VIM]. So it is most important to distinguish carefully the concept of an *"individual measured value"* which is exactly known from the concept of a *"quantity value of a measurand"* which is not exactly known (cf. chapter 2.1).

The validity of the calculated values for the measurement uncertainty is quantified by the so-called "confidence level" (see appendix D). In most cases it is not useful to distinguish between an interval with a confidence level of 95% and e.g. 94% or 96%. It is particularly difficult to justify intervals with a confidence level of 99% and above, even if it is assumed that no systematic influences have been overlooked, since usually only very few information is available about the extreme portions ("tails") of the probability distributions of the input quantities.

In the same context, it is pointed out that rounding rules must be applied to the results in order to avoid the simulation of evaluation results with excessively high accuracy (cf. chapter 4.7.2).

[1] Also see e.g. [EA-4/16], [EA-4/02], [EUROLAB], [EURACHEM], [VDI 2618], [VDI 2622], [ISO 5168], [VDI 2449]

2 Scope

2.1 Measurement uncertainty [2]

The measurement uncertainty can be determined for any measurement result. In the course of a measurement uncertainty study the limits are estimated between which the **true value** of a determined measurement result lies at a specified confidence level (usually 95%).

It is a common misinterpretation to understand measurement uncertainty [VIM, 2.26] in terms of a **measurement error**. A measurement error is defined as a *"measured quantity value minus a reference quantity value"* [VIM, 2.16]. It relates exclusively to a single measured value. It does not relate to the **possible** deviation of the quantity value calculated for a measurand from several individual measured values from the true value of this measurand.

The dispersion of the individual measured values despite seemingly identical measurement conditions is the result of numerous influences which are not controllable by the measuring conditions. These influences can therefore change in an uncontrolled way with each repetition of the measurement.

Deviations of the individual measured values from the median value of their distribution, which are once positive and once negative during repeated measurements, are referred to as **random measurement errors**. If only random measurement errors existed, the median value would be equal to the true value of the measurand. This median value would be obtained as the mean value of the individual measured values if it were possible to repeat the measurement an unlimited number of times, since the standard deviation of the mean value disappears in this limit case.

In practice, only a limited number of repeated measurements is possible. Therefore, a certain dispersion of the mean value remains, and with it a certain lack of knowledge about the true value of the measurand. This ignorance is estimated by means of the **measurement uncertainty**. According to [DIN 1319-1], it is defined as a *"parameter obtained from measurements and which − together with the result of measurement − characterizes the range of values within which the true value of the measurand is estimated to lie"*. In the present context, this definition appears to be more appropriate than the definition according to [VIM, 2.26].

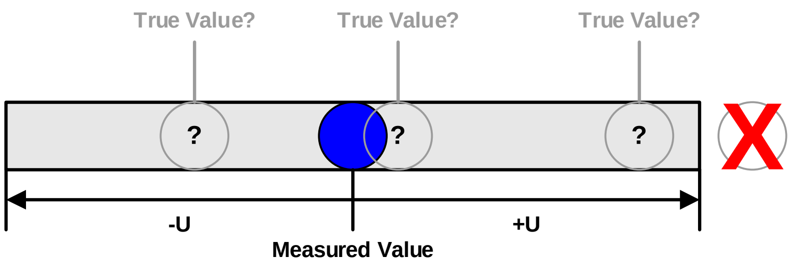


Figure 1: Measurement uncertainty U as a value range for the true value of a measurand

NOTE: The value outside the measurement uncertainty range is not in question as a true value.

In addition to these random measurement errors so-called **systematic measurement errors** occur. They lead to the median value of the distribution of the individual measured values remaining displaced compared to the true value of the measurand even if the measurement were repeated infinitely. As far as possible, identified systematic measurement errors must be minimized, e.g. by adjusting the measuring device or by calculating appropriate correction values. The uncertainty of the correction must be taken into account when determining the measurement uncertainty [GUM, 3.2.3, 3.2.4, 6.3.1, F.2.4.5]. This uncertainty is caused by potentially undetermined systematic measurement errors and any remaining deviations caused by inaccurate correction. These uncertainties must be estimated in an appropriate manner.

[EUROLAB, appendix A.1] contains possible causes of random and systematic measurement errors.

[2] Chapter 2.1 in accordance with [EUROLAB], chap. 2.1, page 10

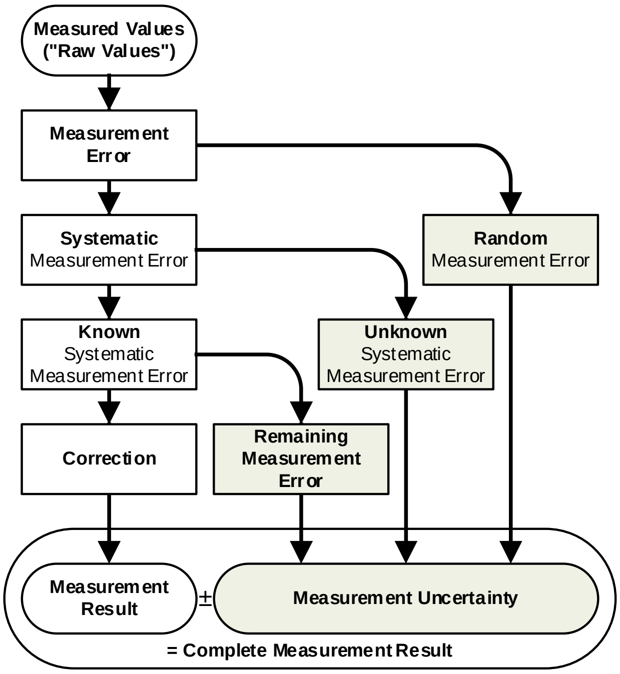


Figure 2: Components of measurement errors and contributions to measurement uncertainty [3]

2.2 Measurement uncertainty and proof of conformity

If the complete measurement result of a characteristic is to be evaluated in terms of specified tolerances, this must be done according to the decision rules of the standard [ISO 14253].

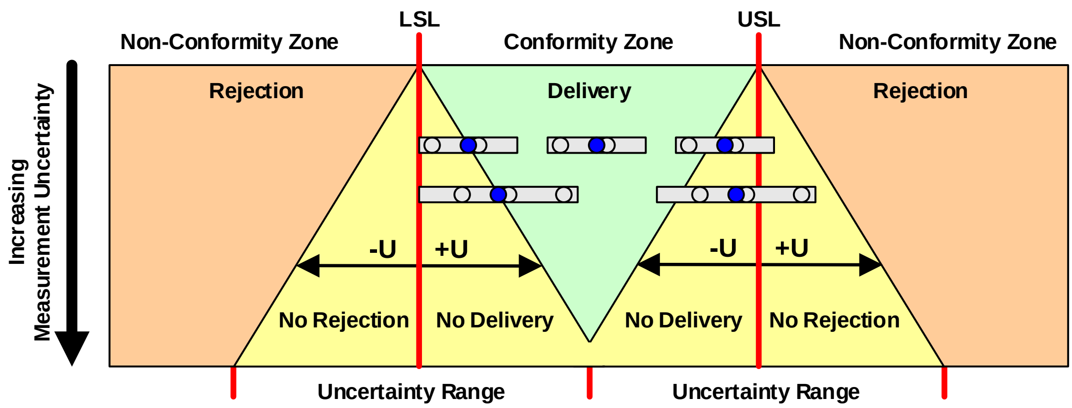


Figure 3: Decision rules according to [ISO 14253]

A conformity zone exists only under the condition *LL + U < UL – U*. This inequality rearranged and *T = UL – LL* substituted yields

$$\frac{2 \cdot U}{T} < 1.$$

With measuring instruments, this ratio should be significantly smaller than 1.

NOTE 1: The previous edition of [VDA-5] referred to the parameter 2U / T as g_{pp} which should not exceed a maximum value G_{pp}. To determine G_{pp}, the range $0.2 \leq G_{pp} \leq 0.4$ was suggested. According to this, in a worst-case scenario, U should amount to no more than 20% of the tolerance T of the characteristic under test. Otherwise, the measuring instrument should be classified as unsuitable for the measuring task. In the current edition of [VDA-5], g_{pp} and G_{pp} are no longer included in this form.

NOTE 2: If a measuring instrument proves to be unsuitable although it represents state-of-the-art, technology, a case of so-called “small tolerances” exists.

[3] Figure 2 in accordance with M. Hernla, QZ **41** (1996), 1156

2.3 Measurement uncertainty and product development

The clarification of the following questions is a typical application of measurement uncertainty as part of product development:

- Evaluation of development progress by reviewing measures for optimizing specific product properties; for this purpose, measurements of characteristics are performed under repeatability conditions e.g. before and after changes are made; the comparison of the measurement results enables conclusions regarding the effectiveness of the measures and may enable assertions regarding adverse effects on the properties of other characteristics.
- Evaluating or determining specifications based on measurement results and their measurement uncertainties.
- Conformity evaluations (see chapter 2.2) for proving that predetermined development objectives have been achieved.

  NOTE 1: Often full specifications of the characteristics are not yet available, but only limit values with which compliance must be proved.

- Carrying out measurements on similar measuring objects under intermediate precision conditions [VIM, 2.22] at different locations (such as at the Bosch and the customer's site) using similar measuring systems and comparing the measurement results.

  NOTE 2: See appendix G regarding the comparability of measuring systems and measurement results.

For comparisons to provide reliable information, the measurement uncertainty must be known in order to evaluate the metrological compatibility of the measurement results (see chapter Definition of terms).

In comparisons, two individual measured values y_1 and y_2 are usually considered to be different if they are at an interval of at least two expanded measurement uncertainties U: $|y_2 - y_1| \geq 2 \cdot U$ (Fig. 4a).

NOTE 3: Different criteria can be determined (e.g. in accordance with appendix G); these criteria must be documented if necessary.

Otherwise the uncertainty ranges of the two values overlap and it is no longer reasonable to exclude that the two measured values might represent the same true value (fig. 4b). The extent of the uncertainty ranges is determined, among other things, by the confidence level (typically 95%). If measurements are exclusively used for the assessment of test results but not for the proof of compliance with agreed or specified properties, a lower confidence level may be acceptable than for production (e.g. 68% instead of 95%) which, however, means a higher risk of an inaccurate evaluation (fig. 4c).

NOTE 4: Because of the increased risk of inaccurate evaluation, specifications and guidelines for testing cannot be derived from measurement results with a reduced confidence level.

NOTE 5: Statements such as "The measurement results correspond within the limits of measurement uncertainty" are frequent conclusions from comparisons. Instead of the correct term "measurement uncertainty", terms such as "error limits", "error tolerances", "error" and "measurement error" are often incorrectly used as synonyms. These terms should not be used in this context in order to make clear assertions.

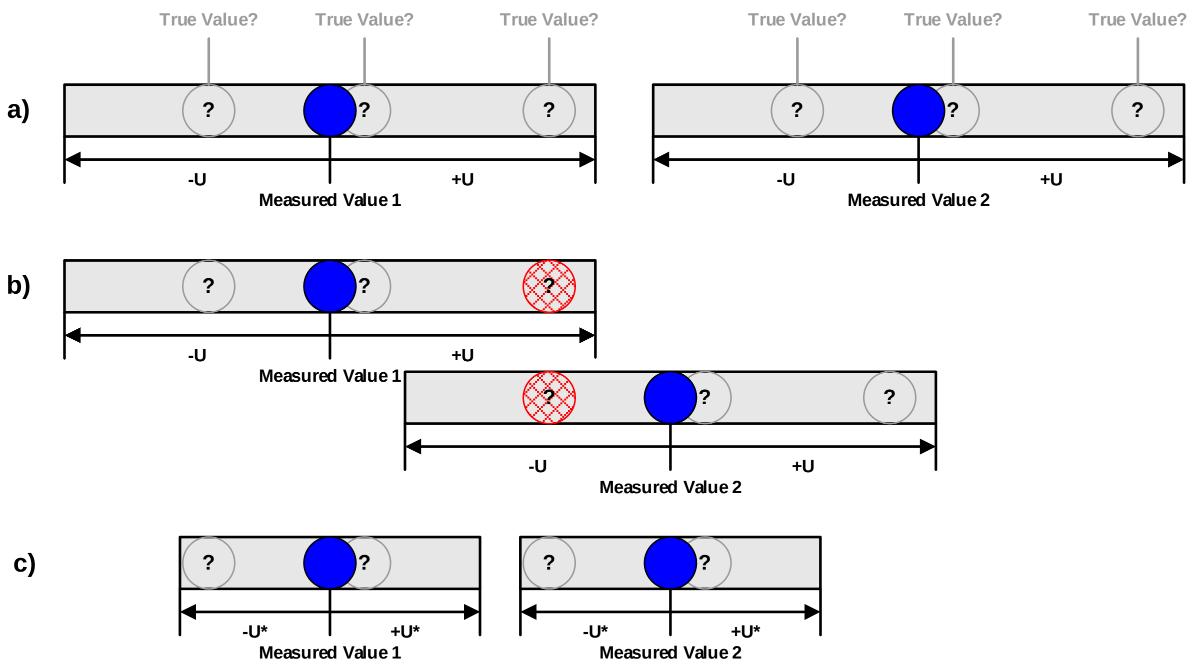


Figure 4: Evaluation of individual readings based on measurement uncertainty

a) *Individual measured values are different at a high level of confidence;*
b) *Hatched value could be the true value of both measured values, therefore no clear difference;*
c) *Individual measured values are different due to lower measurement uncertainty U* < U, but increased risk of inaccurate evaluation since the confidence level is reduced.*

2.4 Measurement uncertainty and production monitoring

In the case of measurements that are needed for production monitoring, a capability study of the measurement process according to [Booklet 10] and evaluating its suitability for the intended measuring task is recommended. This will ensure that the uncertainty of the measurement result is in a reasonable relation to the characteristic tolerance (cf. chapter 2.2 and appendix E). The measured values determined as part of these investigations and any measurement stability monitoring may be used for the calculation of the measurement uncertainty (see chapter 6).

Particularly for production-related application, it is recommended to use preferably data from capability studies and measurement stability monitoring according to [Booklet 10] (see chapter 6). If such data is not available, additive models according to the chapters 4.3.1 and 4.5 can be used which require a relatively low mathematical effort. The applicability of these models must be carefully checked, substantiated and accordingly documented. In case of doubt, usually more complex models have to be used.

Taking account of only those input quantities that are relevant for the case being considered is also recommended. Quantities with little influence on the magnitude of the measurement uncertainty marginally change the calculation result and can be disregarded. This must be carefully checked, substantiated and accordingly documented for every quantity. In case of doubt, the quantity must be taken into account.

2.5 Difference between measurement uncertainty and measuring process capability

As already stated, **measurement uncertainty** provides a value range where the true value for a measurement result can be assumed with a certain level of confidence. However, it does not provide any information about the point within this value range where the true value is most likely to be found, i.e. no probability distribution for the location of the true value of the measurand. Also, the measurement uncertainty is completely independent of any specified tolerances of a characteristic to be measured, i.e. the tolerance T of the characteristic is not included in the measurement uncertainty calculation.

In contrast to this, the **measurement process capability** evaluates the compatibility of the measurement results for a specific characteristic with the tolerance zone of this characteristic, i.e. the position and dispersion of the measurement result within the tolerance zone of the characteristic.

In order to ensure that the measurement results allow for a sufficiently reliable calculation of the statistics C_g, C_{gk} and %GRR and a corresponding classification of the measuring process according to the categories "capable", "conditionally capable" or "not capable", a measurement uncertainty is required that is sufficiently small (see appendix E).

2.6 Range of validity for measurement uncertainty

According to [GUM, 3.1.2] it is mandatory to specify the measurement uncertainty for every complete measurement result. This can lead to the misinterpretation that, in principle, an individual measurement uncertainty study must be made for every measurement performed. However, this is not applicable. Measurement uncertainties are usually determined overall for measurement results of a measurand which are measured under the same conditions.

Even in cases where the measurement uncertainty depends on the quantity value of the measurand, it is not usual to specify an individual measurement uncertainty for every possible measured value. Instead of this, it is possible to divide the relevant measurement range into several ranges. A constant uncertainty is used within each range which is usually the least favorable measurement uncertainty within that range.

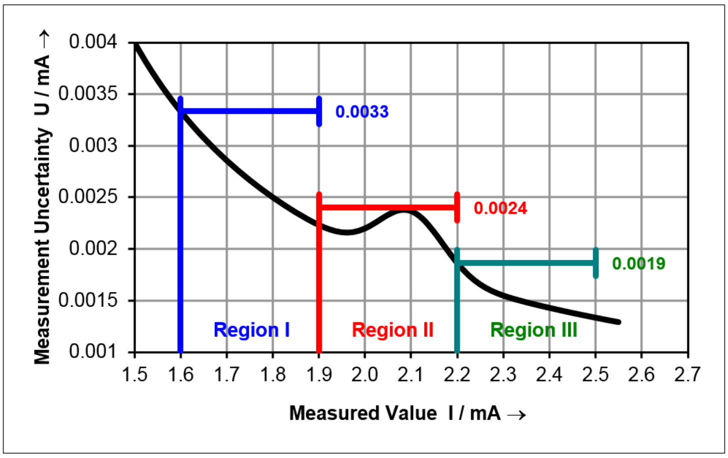


Figure 5: Example for measurement ranges with generally associated measurement uncertainties

3 Flow chart

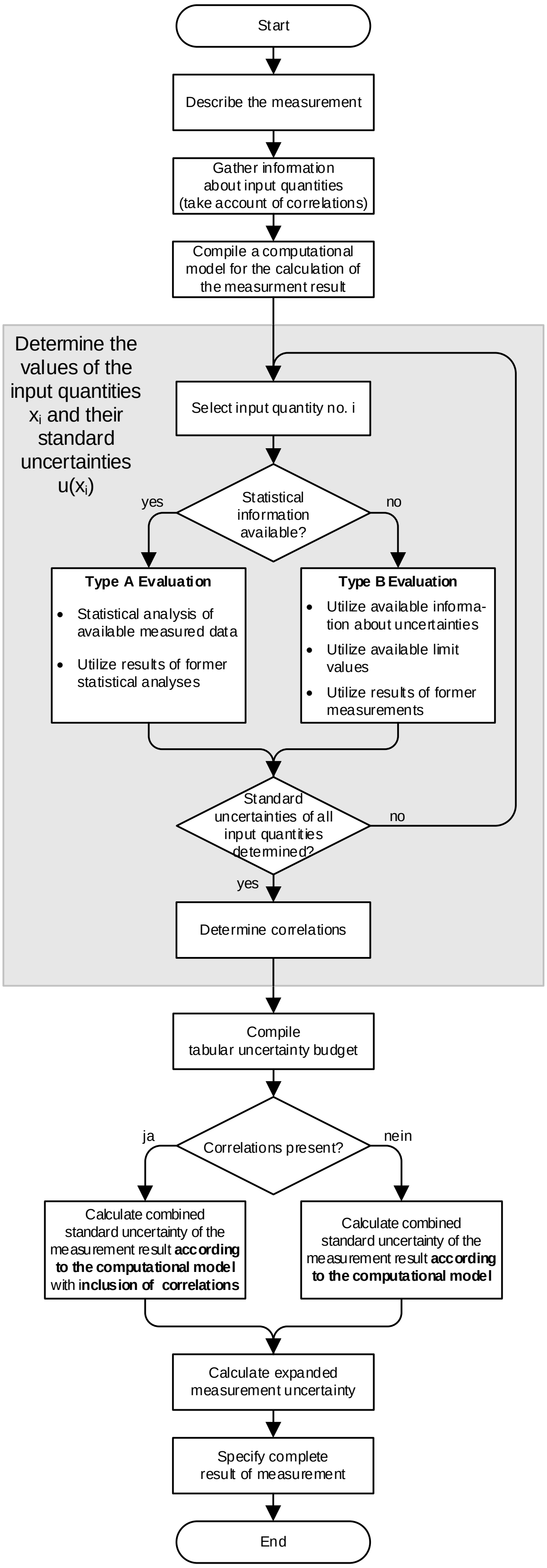


Figure 6: Process flow of a measurement uncertainty study

4 Performing a measurement uncertainty study

This chapter explains the individual process steps that are shown in the flow chart in chapter 3.

4.1 Describing the measurement

[GUM, B.2.5] defines the term "measurement" as a "*set of operations having the object of determining a value of a quantity*". These tasks can be performed either manually or partially or fully automatically. At first, all activities have to be described in detail. Usually, the following information is included:

- Measuring task *(purpose and objective of measurement, such as proof of the conformity of a product characteristic to the specification requirements based on measurement results)*
- Measurand *(characteristic property to be measured, e.g. length, volume, mass, current, resistance, force, power, time, frequency, radiation dose, pH value)*,
- Measurement method *(procedure used for measuring, e.g. measurement of time differences using a stopwatch controlled by light barriers at defined measuring positions and triggered by the measuring object being moved)*,
- Measurement procedure *(description of the measuring principle and its implementation, any explanation of the underlying physical or technical model, e.g. resistance measurement based on current and voltage measurements, speed measurement based on path and time measurements)*,
- Measuring system *(technical design, any measuring position on the measuring object, additional illustrations, diagrams, sketches, description by means of a so-called "measuring circle")*,
- Preparation of the measuring system *(such as heating up)*,
- Workflow description *(such as manual and automatic steps, clamping and releasing or insertion of the measuring object into the measuring system)*,
- Measuring objects *(such as function, specification, tolerances, specified limit values, stability, deviations from provided shape)*,
- State of the measuring object before and possibly after the measurement *(e.g. in case of destructive measurements)*,
- In case of measurement standards the unambiguous identification *(e.g. the ID number)* of the associated calibration certificate and/or reference value, the uncertainty and date of the last calibration, the name of the calibration laboratory,
- Qualitative description of the environmental conditions and general set-up *(e.g. indoor air conditioning)*,
- If necessary for understanding, cross references to physical laws, expected reactions and/or interactions between the measuring system and the measuring object, measurand type *(such as non-repeatable measurement, shear forces)*,
- Information from any existing inspection plans *(e.g. work instructions for inspection or calibration of test equipment)*.

4.2 Gathering information about input quantities

Usually a measurand (output quantity, measurement result) is dependent on several input quantities. Therefore the uncertainty of the measurement result can be determined from the information about the input quantities.

4.2.1 Identifying input quantities

Input quantities are determined systematically (e.g. by means of a cause-effect diagram) and listed in tabular form.

Measuring task:		
Length measurement using a yardstick	Resistance measurement using a multimeter	pH value measurement using a pH meter
Typical input quantities:		
Read length Reading angle Quality of the yardstick Lighting conditions Application set-up Temperature Reference value of the standard Calibration uncertainty …	Current Voltage Frequency Cable length Contact resistance Internal resistance Reference value of the standard Calibration uncertainty …	Difference of potential (ECPD) Temperature Probe material Concentration Liquid composition Measuring principle (device type) Reference value of the standard Calibration uncertainty …

Table 1: Simple examples of measuring tasks with typically associated input quantities

Using the so-called cause-and-effect diagram (see [EQAT], also called an Ishikawa diagram or a fishbone diagram) input quantities can be ordered systematically and combined in groups. Common groups are categories based on 5M such as **m**easuring object, **m**easuring system, **m**ethod, **m**easuring process, **m**an (operator), **m**ilieu (environment) or the categories measurement procedure, measuring object, standard device / calibration.

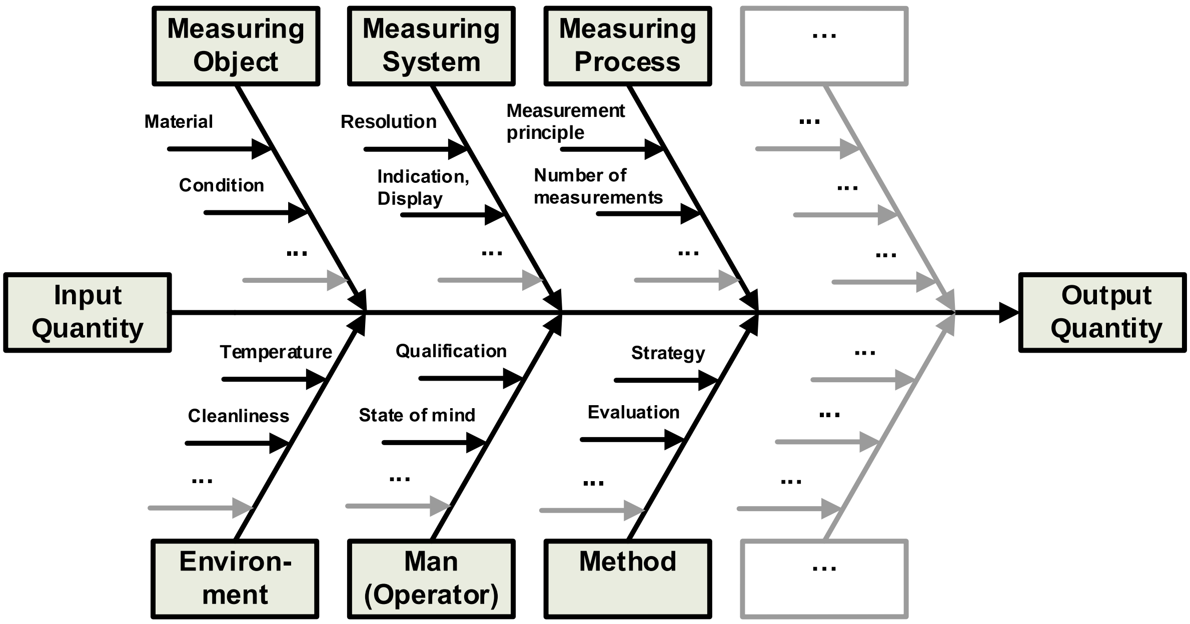


Figure 7: Example of a cause-effect diagram (Ishikawa diagram)

Appendix A contains examples of input quantities of different categories. They can be used as leads for determining input quantities in a particular case.

NOTE 1: Selection and properties of the measuring objects and the inspection personnel can influence the measurement result and thereby the measurement uncertainty (see appendix A). Corresponding input quantities must be taken into account.

NOTE 2: If measurement data from the procedures according to [Booklet 10] is used to determine the measurement uncertainty (see chapter 6), influences from the measuring objects and the inspection personnel including possible interactions are already included in the measurement data and need not be considered separately. Then, however, it is not possible to consider these factors individually and to optimize them since they are not identified as separate input quantities in the uncertainty budget.

4.2.2 Quantifying based on existing information

The necessary quantitative and qualitative information must be obtained for each input quantity to be determined. Information about input quantities can originate from a variety of sources. Typical examples:

- Results of direct measurements,
- Results of previous measurements,
- Experience and subjective evaluations,
- Information from calibration or test certificates,
- Manufacturer's specifications, data sheets
  (including indication of constraints to be considered with the measurement such as humidity, temperature, atmospheric pressure, sensitivity of the measuring instrument, resolution, measurement error, correction values, etc.),
- Measured value dispersion based on experience or repeated measurements
  (e.g. if specifications are unavailable from the manufacturer or other sources),
- Existing measurement uncertainty results that are included in the overall evaluation
  (e.g. from individual devices of the measuring chain),
- Data from investigations of the measuring process capability,
- Information from the preceding measuring chain and/or calibration chain,
- Tabular values or literature values *(e.g. material constants)*,
- Expert forums.

The usability of the information available depends on the type of the input quantities and has to be evaluated under various aspects. Typical examples:

- Temperature, humidity, air pressure,
- The earth's magnetic field, electromagnetic waves *(particularly for electrical quantities)*,
- Stray light *(in particular for optical quantities)*,
- Background radiation *(in particular for radioactive quantities)*.

4.3 Compiling the mathematical model

As already mentioned, a measurand is usually dependent on several input quantities. Therefore the uncertainty of the measurement result can be determined from the input quantities information. Thus, it is necessary to present the relationship in the form of a mathematical model.

This chapter describes a generally valid approach [4] and practice-oriented special cases that can be derived from this approach. To ensure the quickest possible and most direct access to the subject, the special cases are presented first and the general approach is explained at the end of the chapter [5].

The mathematical representation of the model is implemented as a function f depending on the values x_i of the input quantities. This is the so-called model equation from which the value y of the measurand can be calculated:

$$y = f(x_1, x_2, \ldots, x_n) \tag{4.1}$$

with

$x_1, x_2, \ldots, x_n$	Values of the input quantities on which the value y of the measurand depends,
n	Number of input quantities.

*NOTE 1: In this context, these values are referred to as **estimates** of the input quantities and measurands. This is to express the fact that measured values are always affected by uncertainties. In statistics, estimates are represented by lowercase letters while so-called "conventional values" (see chapter Definition of terms) are represented by uppercase letters.*

*NOTE 2: The literature (e.g. [GUM]) often distinguishes between estimates for a quantity (e.g. the measured values for an input quantity) and the quantity itself, to which a **conventional value** is assigned as a quantity value (e.g. the reference value of a standard or the mean value of measured values). Correspondingly, the estimates of the quantity are denoted with lowercase letters and the quantity itself with uppercase letters. This formal distinction is of subordinate importance for practical application. Therefore, this distinction has been dispensed with, i.e. only lowercase letters are used in this booklet. For example, the designation "input quantity x_i" (or simply "input quantity i") is used even if the conventional value is meant, i.e. the input quantity itself, so that the formally correct designation were "input quantity X_i". Instead, the terms "conventional value" or "reference value" are explicitly used whenever a distinction is required.*

*NOTE 3: In addition to measured values x_i of input quantities i which have a direct effect on the measurement result y of the output quantity and which are used to calculate y, other quantities often exist which do not have a direct effect on the output quantity. These indirectly effective quantities are also refered to as **"influence quantities"** [see VIM 2.52]. The distinction is, however, of a more formal nature. Therefore, no distinction is made in this booklet between input quantities and influence quantities, and the term **"input quantity"** is used throughout.*

NOTE 4: The values x_i of the input quantities can have a positive or a negative sign.

*NOTE 5: The use of **SI units** (m, s, Ω, etc.) without a so-called "prefix" denoting decimal multiples or portions (kilo, milli, micro, etc.) is recommended for all quantities. In this case the model equation allows for a simple and efficient dimensional control in order to prevent errors, i.e. the measuring units of the input quantities substituted in the model equation must provide the measurement unit of the output quantity (possibly after algebraic transformation).*

[4] "Generally valid" as far as linear approaches are applicable, i.e. the Gaussian error propagation law

[5] In practice, model equations can contain submodels that correspond to one or more of the model approaches described below (see appendix J.8, for example)

4.3.1 Additive model

In many cases the model function consists of the sum of two or more input quantities:

$$y = x_1 + x_2 + \ldots + x_n \tag{4.2}$$

This model approach requires all x_i input quantities to be used consistently and uniformly in the measurement unit of the output quantity y (see quantity dimension [VIM, 1.7]).

EXAMPLE 1: The total resistance R (measurand) of two resistors R_A and R_i connected in series (input quantities) is calculated according to the model equation $R = R_A + R_i$. The working resistance R_A was measured at 15 kΩ, the internal resistance R_i of the measuring instrument is specified at 100 mΩ. It is essential to ensure that both values are used in the same unit of measurement in the model equation, e.g. R_A = 15 kΩ and R_i = 0.0001 kΩ or R_A = 15,000 Ω and R_i = 0.1 Ω.

EXAMPLE 2: The velocity v (measurand) is made up of the velocity components v_1 and v_2 (input quantities), i.e. the model equation $v = v_1 + v_2$ applies. The values v and v_1 are in km/h while the value v_2 is in m/s. Before application in the model equation, it is therefore necessary to convert either v and v_1 into m/s (1 km/h = 1000 m / 3600 s ≈ 0.278 m/s) or v_2 into km/h (1 m/s = 3.6 km/h).

The additive approach also can be used to determine measurement uncertainties in line with [GUM] in case it is not possible to derive the relation between the input quantities and the measurement result in the form of an equation from physical models because of its complexity. A prerequisite is that the deviations from the conventional values of the input quantities are quantifiable (see chapter 4.2.2) and independent from each other (see chapter 4.4.3). In these cases, a model equation is formulated in the form

$$y = y_0 + \delta x_1 + \delta x_2 + \ldots + \delta x_n \tag{4.3}$$

with

y_0	conventional value for the measurement result y (no uncertainty), often estimated by correcting the indication y‘ (cf. chapter 4.3.3);
$\delta x_1 \ldots \delta x_n$	deviations from the conventional value of the input quantities in the measuring unit of the measurement result; expected value 0; $1 \le i \le n$.

Application examples: see appendix J (except J.7)

4.3.2 Multiplicative model

In some cases, the model function consists of a product and/or quotient of two or more input quantities:

$$y = \frac{x_1 \cdot x_2 \cdot \ldots}{\ldots \cdot x_{n-1} \cdot x_n} \tag{4.4}$$

This approach requires all input quantities x_i to be used in measurement units whose composition as a product or quotient according to the model equation gives the measurement unit of the output quantity y. When using relative units such as %, chap. 4.4 (see note and example) has to be taken into consideration.

EXAMPLE 1: The resistance R (measurand) is determined by measuring the voltage U and the current I (input quantities), i.e. the model equation R = U / I applies. The values U = 6 V and I = 12 mA are measured. The resistance R is specified at 500 Ω. Because 1 Ω = 1 V/A applies, the current I must be converted into A before the model equation is used, i.e. I = 0.012 A.

EXAMPLE 2: The velocity v (measurand) is determined by measuring the distance travelled s and the time required t (input quantities), i.e. the model equation v = s / t applies. The measured distance is specified as s = 100 m, while the measurement result for the time required is t = 14.9 s, so that v = 6.7114 m/s results. The speedometer is calibrated in mph (miles per hour) and shows the velocity v = 15 mph. Before application in the model equation, it is therefore necessary to convert v into m/s (1 mph = 0.44704 m/s), i.e. to use v = 6.7056 m/s (recommended). Alternatively, s could be converted into miles and t into hours (not recommended, since SI units are not used consistently).

NOTE: Conversion factors (and natural constants) must be considered to be constants without uncertainty. However, if these quantities are rounded, this inaccuracy (cf. chapter 4.5) must be taken into account properly (cf. chapter 4.7.2).

Application examples: see appendix J.1.3 and appendix J.1.4.

4.3.3 Linear function

In certain cases, the relation between the output quantity y and one or more input quantities x_i can be described using the following model:

$$y = (a_1 + b_1 \cdot x_1) + (a_2 + b_2 \cdot x_2) + \ldots + (a_n + b_n \cdot x_n) \tag{4.5}$$

with the constants a_i and b_i, $1 \le i \le n$.

NOTE 1: In the special case n = 1 Eq. (4.5) represents a straight line with intercept a_1 and slope b_1.

A common application is the (mathematical) correction of measurement results. The indication of a measuring instrument provides a measured value y' which is subject to a correction $K(y')$ due to a known systematic influence (such as temperature). Then, the corrected measurement result can be calculated as follows (see appendix F):

$$y = y' + \underbrace{\alpha_K + \beta_K \cdot y'}_{=K(y')} \tag{4.6}$$

with

y	corrected measurement result *(often utilized as conventional value y_0)*,
α_K	correction constant *(intercept of the correction function)*,
β_K	correction factor *(slope of the correction function)*,
y'	uncorrected measurement result *("raw value")*.

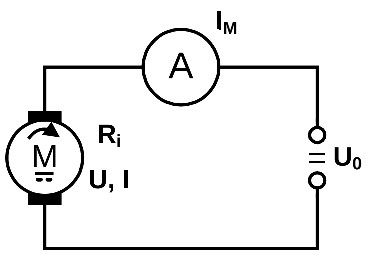
NOTE 2: Eq. (4.6) is often used as a submodel for the conventional value y_0 in the overall model (see e.g. the model equation used in appendix J.3). A use case that is important in practice is using Eq. (4.6) in the form $y_0 = y' + K$ with $\beta_K = 0$ and K calculated as the difference of the reference value y_0 of the standard and of the uncorrected measurement result y': $K = y_0 - y'$. In case of several results y' with the same standard, the mean value $\overline{y'}$ is used.

Application examples regarding correction: see appendix J.2, J.3 and J.8.

4.3.4 General case

A generally applicable approach is inherently incomplete and cannot be described in full. The approach also places greater demands on the physical and mathematical understanding of the user. The essential approach is based on physical laws from which the model equation is derived.

This is explained using the very simple example of an electrical power measurement. The power consumption P of an electrical DC engine must be determined based on the measured current I_M and the internal resistance R_i specified for the engine (e.g. in the manufacturer's data sheet). This means that the input quantities I_M and R_i are used in this case to determine the measurand P. Correspondingly, the general model equation $y = f(x_1, x_2, \ldots, x_n)$ is applied in the form



$$P = f(I_M, R_i) \tag{4.7}$$

According to fundamental physics, the following applies to the electrical power consumption of the engine:

$$P = U \cdot I \tag{4.8}$$

U represents the voltage drop across the engine while I represents the current through the engine. Ohm's law provides the relationship between U and R_i:

$$U = R_i \cdot I \tag{4.9}$$

In the circuit shown the following applies to the current I as per Kirchhoff's current law:

$$I = I_M \tag{4.10}$$

U and I substituted yields the model equation:

$$P = R_i \cdot I_M \cdot I_M = R_i \cdot {I_M}^2 \tag{4.11}$$

Application examples: see appendix J.7 and appendix J.8.

4.4 Input quantities: Determining the quantity values and standard uncertainties

The model equation allows the measurement result y to be calculated from known values x_i of the input quantities (cf. chapter 4.3). The measured value y is always affected by an uncertainty $u_C(y)$. If the uncertainties $u(x_i)$ of the input quantities x_i are known, the uncertainty $u_C(y)$ of the measured value y also can be determined using the model equation.

[GUM] standardized the determination of measurement uncertainty at international level. The determination methods have been adopted accordingly in this guide. [GUM] distinguishes between the two following methods for determining the input quantities x_i and their standard uncertainties $u(x_i)$:

- **Type A evaluation** (method A)**:** The values x_i and $u(x_i)$ are determined based on repeated measurements and the statistical analysis of these measurements.

  EXAMPLES: Data measured for the determination of measurement uncertainty; results of stability monitoring; records of previous investigations.

- **Type B evaluation** (method B)**:** The values x_i and $u(x_i)$ are determined based on other sources and the processing of these.

  EXAMPLES: Manufacturer's specifications; limit values; parameters known from previous investigations; values from literature.

The appropriate approach for determining the values x_i and the standard uncertainties $u(x_i)$ of the input quantities results from the accuracy requirements, the available measurement equipment and economic considerations. Either a type A or a type B evaluation must be applied to each input quantity. Using the same method for all input quantities is not a requirement (see examples in appendices J.3, J.6, J.7 and J.8). Procedures and calculation steps always have to be documented.

NOTE: In metrology "accuracy specifications" are often given relative to a specific reference value, e.g. as a percentage of the full scale value of the measuring range. Experience has shown that specifications of this type are a common source of error since it is not recognized that the <u>absolute</u> value of the uncertainty is actually given which applies to the entire measuring range. The percentage applies at the reference point only. It does not apply to any other point of the remaining measuring range.

EXAMPLE: The uncertainty of a pressure cell with a measuring range of 0 to 10 bar is specified as 0.5% of the full scale value, i.e. 10 bar. This specification is equivalent to the absolute value of 0.05 bar which applies to the entire measuring range from 0 to 10 bar. For a measured value of e.g. 0.4 bar, a relative uncertainty of 0.05 bar/0.4 bar = 0.125 results, i.e. 12.5%.

4.4.1 Type A evaluation

4.4.1.1 Determination from latest measurement results

Measurements of the input quantities i are performed under defined measurement conditions which must be documented. Conditions that are to be expected later during the use of the measuring system should be realized as far as possible. The value x_i is estimated by means of the **arithmetic mean value**

$$\overline{x}_i = \frac{1}{m} \cdot \sum_{k=1}^{m} x_{ik} \tag{4.12}$$

of the m individual measured values x_{ik} [GUM, 4.2.1]. It is assumed here that a normal distribution can be supposed which is usually acceptable. The number m of the individual measured values must be sufficiently large to ensure a reliable quantity value x_i. A quantitative measure for this "reliability" is the so-called **confidence level** (see appendix D).

NOTE 1: The better the measurement conditions meet the repeatability conditions, the more reliable the statistical assertions. So, defined measurement conditions have to be seen as measurements which are preferably performed

- *using the same measuring system (measuring instrument),*
- *the same measuring objects,*
- *and the same measurement procedure*
- *under the same, stable conditions*
- *carried out by the same operator*
- *at the same location*
- *within a short time interval.*

If there are doubts as to whether the measurement conditions are appropriate, correlations of the input quantities have to be investigated by means of parametric studies (see appendix C) and corrections of the measured values have to be made as appropriate (see appendix F). Alternatively, it should be checked whether a type B evaluation could lead to more reliable results and therefore should be used (cf. chapter 4.4.2).

Random influences during measurement of the input quantity i cause a **dispersion of the individual measured values** x_{ik} which are best described by their **empirical standard deviation**

$$s(x_i) = \sqrt{\frac{1}{m-1} \cdot \sum_{k=1}^{m} (x_{ik} - \bar{x}_i)^2} \tag{4.13}$$

around their mean value $\bar{x}_i$ [GUM, 4.2.2].

The **standard uncertainty** of the input quantity i is described by the **dispersion of the mean value** $\bar{x}_i$

$$u(\bar{x}_i) = \frac{s(x_i)}{\sqrt{m}} \tag{4.14}$$

[GUM, 4.2.3].

*NOTE 2 [6]: The applicability of Eq. (4.14) with m > 1 assumes **mandatorily** that the estimate for the conventional value x_i of the input quantity i is determined as a mean value $\bar{x}_i$ from m > 1 measured values x_{ik}, which represent individual observations of the input quantity i that are statistically independent of each other, i.e. uncorrelated.*

- *Correlations between the individual values of a data series exist if e.g. differences between the individual measured values of the data series do not vary randomly, but are constant or change systematically (see also chapter 4.4.3). In case of doubt, appropriate data analyses must be performed (see appendix C). Otherwise m = 1 has to be used, i.e. the standard deviation of the individual measured values x_{ik} is used as the standard uncertainty.*
- *A measurement uncertainty that is determined based on mean values must only be applied to mean values obtained from the same number of individual measured values in the subsequent use of the measuring system. This condition is often disregarded in practice.*

EXAMPLE: Instead of individual measured values, a measuring system shows the mean value of a defined number of individual measured values as the "measured value". The number of averaged individual measured values is determined by the setting of the sampling time.

- *For the result of the measurement uncertainty study the number of individual measured values averaged and output as a single "measured value" is not decisive. However, the number of averaged "measured values" included in the uncertainty evaluation is decisive (m = 1 for one "measured value", m > 1 for several "measured values").*
- *The result of the measurement uncertainty study is only applicable to subsequent measurement results on the condition that the measuring system works with the same parameter settings as those used during the measurement uncertainty study (e.g. integration time, sampling frequency).*

Required measuring system settings and the measurement procedure to be used must be precisely defined and documented (e.g. in a test or work instruction).

[6] In accordance with [EUROLAB], appendix A.5 (page 44) and appendix A.6 (page 47)

4.4.1.2 Determination from former measurement results

The concept of standardized measurement uncertainty allows results of previous measurements to be used to determine measurement uncertainty [7]. This is advantageous in practice if only a few measurements of a certain input quantity i can be performed for technical or economic reasons, so that too few individual measured values are available to determine a sufficiently reliable value for the dispersion from their standard deviation. In this case, results from former measurements can provide more reliable conclusions which, for example, are available as a **"pooled" standard deviation s_p** [GUM 4.2.4].

If, instead of $\mathbf{s_p}$, the results of several measured data sets are available for an input quantity i, i.e. the standard deviation $s_j(x_i)$ and the number m_j of the individual measured values x_{ijk} are known for each data set j whereas the values x_{ijk} are unknown, the pooled standard deviation s_p is determined according to the following calculation rule [GUM, H.3.6; ISO 5725-2, 7.4.5.1]:

$$s_p(x_i) = \sqrt{\frac{\sum_{j=1}^{j_p} (m_j - 1) \cdot s_j^2(x_i)}{\sum_{j=1}^{j_P} (m_j - 1)}} \tag{4.15}$$

with

j_P	number of pooled data sets,
m_j	number of measured values in data set no. j,
$s_j(x_i)$	standard deviation of data set no. j for input quantity no. i.

It is important to note that previous results for $\mathbf{s_p}$ are only usable provided that date and time and parameters of the former measurements have a negligible influence on the input quantities. In principle, conditions that are similar to those encountered during practical use of the measuring system must be created when determining the measured values. Usually a qualified evaluation of this requirement can only be made using the documentation of the earlier measurement uncertainty study as a basis.

The associated standard uncertainty is calculated according to

$$u(\bar{x}_i) = \frac{s_p(x_i)}{\sqrt{m}} \tag{4.16}$$

In this calculation $m \geq 1$ represents the number of individual measured values x_{ik} which were actually measured to determine the value x_i of the input quantity i in the course of the **current** measurement uncertainty study (rather than the number of all previously determined individual measured values that have contributed to $\mathbf{s_p}$) [GUM H.3.6].

NOTE: With regard to the applicability of m > 1, note no. 2 in chap. 4.4.1.1 must be considered.

4.4.2 Type B evaluation

The standard uncertainties of input quantities can be determined even if multiple observation is not possible so that a type A evalution is not applicable. These include the following cases in particular:

- It is not possible to perform measurements (e.g. for technical or economic reasons).
- Measurements were performed previously, however, only the evaluation results are available (e.g. dispersion, distribution, unless used according to chap. 4.4.1.2) [7].
- Input quantities cannot be determined metrologically (e.g. in case of subjective influences, see appendix A).

In such cases the **results** of former investigations or existing experience can be utilized to estimate the value range to be expected for the input quantities and the distributions they can be assigned to.

According to [GUM, 4.3.1] standard uncertainties can be obtained from

- the evaluation results of former measurements [7],
- experience or general knowledge about behavior and properties of the relevant materials or measuring instruments,
- manufacturer's specification and data sheets,
- data provided in calibration certificates and other certificates,
- uncertainties of reference data taken from handbooks.

The requirements of the model (cf. chapter 4.3) and the practical experiences of the measurement engineer are decisive factors for selecting the data sources which are reasonably utilized.

Data obtained from interlaboratory tests provide excellent conditions for a type B evaluation. These data are particularly utilized for measurement procedures where, because of complex interactions, only the overall procedure can be evaluated rather than the individual contributions of existing influences (see [ISO 21748] for more information).

4.4.2.1 Determination using available uncertainty data

A distinction should be made between the following cases:

- If the uncertainty data is specified as a multiple of a standard deviation, the standard uncertainty is calculated by dividing the available value by this multiplier [GUM, 4.3.3].
- If a confidence level is specified for the uncertainty data (e.g. 90%, 95% or 99%), a normal distribution can be assumed. The standard uncertainty is calculated by dividing the available value by the corresponding coverage factor k_p (e.g. 1.64, 1.96 or 2.58; see appendix D) [GUM 4.3.4].

  NOTE: It is also assumed that sufficient degrees of freedom ($\nu \geq 20$) were available so that the approximation $\nu \to \infty$ is sufficiently met (see appendix D).

- If the uncertainty data is shown to be an expanded measurement uncertainty and the confidence level is not specified, the standard uncertainty is calculated by dividing the available value by $k_p = 2$ (corresponding to the confidence level of 95.45%, see appendix D).
- Uncertainty data from available sources (such as data sheets and literature) are applied unchanged as standard uncertainty unless further information about contributions and components is available and the uncertainty is not explicitly designated as an expanded measurement uncertainty.

4.4.2.2 Determination using available limit values

It is assumed that the available limit values a_- and a_+ were determined based on measured values x_i which belong to a statistical distribution and lie with a certain probability within the range between a_- and a_+. The mid-point $(a_+ + a_-)/2$ of this range is at a distance of $a = (a_+ - a_-)/2$ from these limits.

- If the distribution and the confidence level are known and included in Table 2, the standard uncertainty $u(x_i)$ is determined according to the corresponding calculation rule in Table 2.
- If corresponding data is missing, the information in Table 2 can be used to select an appropriate distribution.

[7] [GUM] does not provide a clear criterion for assigning the utilization of data from previous studies to a type A or a type B evaluation. The present guideline primarily assigns such data to type A (see [GUM, 4.2.4]). This does not mean that the assignment to type B cannot be equally reasonable (see [GUM, 4.3.1]). Evaluation results are not influenced by this assignment.

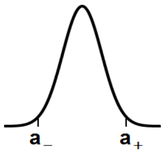
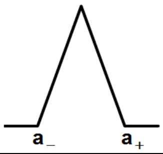
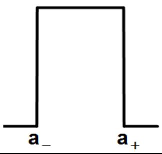
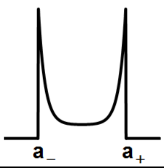
Distribution (density function)	**Information about the measured values x_i**	**Position of the measured values x_i within the limits a_- and a_+**	**Confidence level (probability) for the position of the measured values x_i within the limits a_- and a_+**		**Standard uncertainty $u(x_i)$**
Normal distribution 	Values are random	Pooled around central position	Assumption of a probability of less than 100% is reasonable and necessary	99.73%	$u(x_i) = a/3$ [GUM, G.1.3]
				95.45%	$u(x_i) = a/2$ [GUM, G.1.3]
Triangular distribution 	Values are random	Pooled around central position	All values within the limits a_- and a_+ (e.g. for physical reasons)	100%	$u(x_i) = a/\sqrt{6}$ $(\sqrt{6} \approx 2{,}45)$ [GUM, 4.3.9]
Uniform or rectangular distribution 	None	Unknown			$u(x_i) = a/\sqrt{3}$ $(\sqrt{3} \approx 1{,}73)$ [GUM, 4.3.7]
U-distribution 	None	Pooled close to the limits			$u(x_i) = a/\sqrt{2}$ $(\sqrt{2} \approx 1{,}41)$

Table 2: Distributions for input quantities with calculation rules for the standard uncertainties

Examples of practical applications:

- Normal distribution: Results of statistical analyses (e.g. measured values determined under repeatability conditions); calibration certificate data (e.g. reference value).
- Triangular distribution: Interpolated values of input quantities; special measuring systems (e.g. Wheatstone bridge circuit with compensation as a zero point detector); approximation of normal distribution.
- Rectangular distribution [8]: Results from which only limit values are known; results arising from digitization.
- U-distribution: Sine-wave-like oscillations, measurement results with hysteresis.

Depending on the application, other distributions are required (e.g. trapezoidal distribution, modal distribution). If necessary this must be tested and justified in each case.

Unless it is ensured in case of triangular, rectangular or U-distributions that all measured values lie within the limits a_- and a_+ (confidence level < 100%), different calculation rules apply to the standard uncertainties. The technical literature should be referred to for this point.

[8] Worst case leading to the maximum possible contribution of this input quantity to the overall uncertainty

4.4.3 Correlated input quantities

If a change of the input quantity i also causes a change of the input quantity j and vice versa, these input quantities are correlated. Correlations are generally expected when two quantities depend on each other or on a common third (possibly hidden) quantity, or on several such quantities. [9]

- This dependence can relate directly to the physical quantities. This means that e.g. the relative mass fractions of the constituents of a mixture of substances are dependent on each other, since their sum is equal to one. This is true regardless of changes of the relative portions which e.g. result from chemical changes within the mixture. [9]
- Physical quantities are often independent of each other, however, their values are not determined independently of each other. This is the case if two quantities are determined in the same experiment – such as intercept and slope of a calibration curve – or if the same standard is used for different input quantities. Further typical examples are shared influences of measuring parameters (such as temperature on thermal expansion) and temporal influences on different input quantities (such as temporally different warming-up of the measuring instruments used). Then, the determined quantities depend on shared quantities: the calibration data set or the reference value of the standard. [9]

Taking into account correlations complicates mathematical work considerably (see appendix C). Thus, it is avoided as far as possible. Correlations are typically negligible

- if the data sets originate from different experiments which are independent of each other and which were carried out at different times,
- if constant input quantities are present (i.e. in case an input quantity does not change, this input quantity cannot have an effect on another input quantity even if these quantities are correlated),
- if the standard uncertainty of one of the two input quantities is negligible (see appendix C.1, NOTE NOTE 5).

If non-negligible correlations exist, the detailed analysis and more complex mathematical processing often can be avoided if the model considers parameters affecting several input quantities as additional and independent input quantities with an independent standard uncertainty (such as ambient temperature).

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[9] In accordance with [EUROLAB], appendices A.5 and A.6

4.5 Calculating the combined standard uncertainty

*NOTE 1: The basis of the following calculation rules – including the general case – is the Gaussian error propagation law, i.e. a **linear** approximation. This is based on the expansion of the model equation into a Taylor series which is discontinued after the linear term. In special cases (e.g. in case of high precision inspections), it may be necessary to take into account the square term or even higher terms of the Taylor expansion. The appropriate literature should be referred to for this purpose.*

*NOTE 2: All calculation rules below assume **uncorrelated** input quantities.*

Model	Model equation	Combined standard uncertainty $u_C(y)$ of the measurement result y
Additive (chap. 4.3.1)	$y = x_1 + x_2 + \ldots + x_n$	$u_C(y) = \sqrt{u^2(x_1) + u^2(x_2) + \ldots + u^2(x_n)}$ (4.17)
	$y = y_0 + \delta x_1 + \delta x_2 + \ldots + \delta x_n$	$u_C(y) = \sqrt{u^2(\delta x_1) + u^2(\delta x_2) + \ldots + u^2(\delta x_n)}$ (4.18)
Multiplicative (chap. 4.3.2)	$y = \dfrac{x_1 \cdot x_2 \cdot \ldots}{\ldots \cdot x_{n-1} \cdot x_n}$	$\dfrac{u_C(y)}{y} = \sqrt{\left(\dfrac{u(x_1)}{x_1}\right)^2 + \left(\dfrac{u(x_2)}{x_2}\right)^2 + \ldots + \left(\dfrac{u(x_n)}{x_n}\right)^2}$ (4.19)
Linear function (chap. 4.3.3)	$y = a_1 + b_1 \cdot x_1 + \ldots$ $\ldots + a_n + b_n \cdot x_n$	$u_C(y) = \sqrt{\begin{matrix} u^2(a_1) + x_1^2 \cdot u^2(b_1) + b_1^2 \cdot u^2(x_1) + \ldots \\ \ldots + u^2(a_n) + x_n^2 \cdot u^2(b_n) + b_n^2 \cdot u^2(x_n) \end{matrix}}$ (4.20)
General (chap. 4.3.4)	$y = f(x_1, x_2, \ldots, x_n)$	$u_C(y) = \sqrt{c_1^2 \cdot u^2(x_1) + c_2^2 \cdot u^2(x_2) + \ldots + c_n^2 \cdot u^2(x_n)}$ (4.21) with the sensitivity coefficients $c_i = \dfrac{\partial y}{\partial x_i}$

with

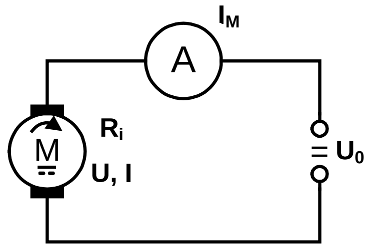
$u(x_i)$	standard uncertainty of the values x_i of the input quantities i with $1 \le i \le n$,
$u(\delta x_i)$	standard uncertainties of the deviations δx_i from the expected values x_i of the input quantities i with $1 \le i \le n$,
$u_C(y)$	combined standard uncertainty of the measurement result y,
y	measurement result (corrected if necessary).

Details of the derivation of Eqs. (4.17) to (4.21) are given in appendix B, application examples are given in appendix J.

NOTE 3: In case of the multiplicative model the combined <u>relative</u> standard uncertainty $u_C(y) / y$ of the measurement result y can be directly determined as the geometric sum of the given <u>relative</u> standard uncertainties $u(x_i) / x_i$ of the input quantities x_i.

Example

NOTE 4: The general case usually places greater demands on the physical and mathematical understanding of the user.

General case	Example of power measurement according to chap. 4.3.4
	
Model equation $y = f(x_1, x_2, \ldots, x_n)$	$P = P(R_i, I_M) = R_i \cdot I_M{}^2$
Sensitivity coefficients $c_i = \frac{\partial y}{\partial x_i}, \quad 1 \le i \le n$	$c_{R_i} = \frac{\partial P}{\partial R_i} = \frac{\partial}{\partial R_i} R_i \cdot I_M{}^2 = I_M{}^2$ $c_{I_M} = \frac{\partial P}{\partial I_M} = \frac{\partial}{\partial I_M} R_i \cdot I_M{}^2 = 2 \cdot R_i \cdot I_M$
Combined standard uncertainty $u_C(y) = \sqrt{c_1^2 \cdot u^2(x_1) + c_2^2 \cdot u^2(x_2) + \ldots + c_n^2 \cdot u^2(x_n)}$	$u_C(P) = \sqrt{c_{R_i}^2 \cdot u^2(R_i) + c_{I_M}^2 \cdot u^2(I_M)}$ $= \sqrt{I_M{}^4 \cdot u^2(R_i) + 4 \cdot R_i^2 \cdot I_M{}^2 \cdot u^2(I_M)}$ $= \sqrt{I_M{}^2 \cdot u^2(R_i) + 4 \cdot R_i^2 \cdot u^2(I_M)} \cdot I_M$

4.6 Expanded measurement uncertainty

The expanded measurement uncertainty U is a parameter which identifies a range around the measurement result that can be expected to include a large proportion of the distribution of the values that could reasonably be assigned to the measurand [10]. It is calculated as

$$U = k_p \cdot u_C \qquad (4.22)$$

with

u_C combined standard uncertainty (cf. chapter 4.5),
k_p coverage factor for a specific confidence level.

The factor k_p used (or alternatively the confidence level) must be documented.

NOTE 1: In metrology a confidence level of 95.45% is preferably used which corresponds to k_p = 2. This implies m $\geq$ 20 measured values (see appendix D).

NOTE 2: The value of k_p is not only determined by the confidence level but also by the degrees of freedom. The degrees of freedom are relevant in particular if (considerably) less than 20 measured values are available, or if an optimal selection of k_p is required (e.g. if it is essential to avoid excessive measurement uncertainty specification). For further details, see appendix D.3.

If type A evaluation is used (exclusively), the degrees of freedom always must be specified [GUM 4.2.6].

NOTE 3: Alternatively, the number of measured values can be specified instead of the degrees of freedom.

[10] According to [GUM, 2.3.5] and [VIM(2), 3.9]

4.7 Complete measurement result

4.7.1 Notation

The complete measurement result of a measurand is made up of the measured value y, corrected as necessary, and the associated expanded measurement uncertainty U. The following notations can be used:

- y ± U (recommended for Bosch)
- y, U
- y, U_{rel}
- y (1 ± U_{rel})
- y (U) (not recommended)

Here, U_{rel} denotes the expanded measurement uncertainty related to the measured value: $U_{rel} = U / |y|$.

NOTE 1: Notations such as 5 mA + 5% are not permitted.

The range within which the conventional value of the measurement result is expected is given by the limits y – U and y + U.

NOTE 2: In the case of unilaterally limited characteristics, it is possible for y - U to fall below the value 0. If this is the case, the range from 0 to y + U applies to the conventional value.

NOTE 3: If corrections have been calculated and applied when determining the measurement uncertainty, it is useful in many cases to specify these corrections separately as an additional information (examples: see appendix J.3, page 80; appendix J.8, page 116).

NOTE 4: If several measurement results are available (no individual value), presenting data in tabular form is permitted.

4.7.2 Rounding rules

According to [GUM, 7.2.6] the numerical values for the measurement result y and its expanded measurement uncertainty U may not be specified with an excessive number of digits. A maximum of two significant decimal places[11] is usually sufficient to specify U. In some cases, it may be necessary to retain additional digits in order to prevent rounding deviations in subsequent calculations.

NOTE 1: Unlike the rounding of the final result, the rounding of intermediate results and the values of input quantities should be avoided as far as possible.

Correlation coefficients must be specified to three significant decimal places if their absolute values are close to one.

It makes no sense to specify the values for the measurement result y and its expanded measurement uncertainty U in the final result with more than one additional decimal place compared to the resolution of the measuring system. More decimal places cannot be recorded with the measuring instrument being used, and are therefore worthless.

The final results of uncertainty calculations must be rounded up. Example: U = 0.422 µm is rounded up to U = 0.43 µm. Results of degree of freedom calculations (see appendix D.3) must be rounded down to integers.

NOTE 2: However, common sense should always prevail so that marginal cases such as U = 0.4205 µm are rounded down to U = 0.42 µm instead of rounding up to U = 0.43 µm.

[11] Digits of a number are referred to as “significant digits” if the corresponding number can be considered as lying within the limits of the deviation of the least-significant digit (see ISO 80000-1:2009 + Cor 1:2011). *Example: The numerical value 4.12 has 3 significant digits if the exact value is within the range 4.115 < x < 4.125, since all values in this range give the result 4.12 when rounded according to customary rules.*

4.8 Tabular uncertainty budget

The required work steps for determining and specifying the measurement uncertainties are described in the preceding subchapters of chapter 4. A comprehensible documentation of these work steps must be compiled for each specific case of application. No binding format is specified for this documentation. However, creating an uncertainty budget in tabular form is recommended. Appendix I contains a suggestion for a tabular presentation of this type which is also used for the examples in appendix J. Supplementary descriptions are required in most cases with texts and images for the measuring task, the measurement setup and the selection of input quantities and calculations.

NOTE 1: Unfortunately, the English term "budget" often results in mistakable terms when translated into other languages. Actually it is mainly a consistent listing of contributions to uncertainty, i.e. a sort of "balance-sheet".

For measuring uncertainties of variable quantities (e.g. characteristic curves) that are determined at several reference points (parameter settings), the tabular presentation becomes more complex as the number of reference points increases (e.g. if there is one table for each reference point). In this case, curves or arrays of curves are used in practice in order to provide the measurement uncertainty in dependence of selected parameters.

4.8.1 Minimum requirements for documentation

A tabular uncertainty budget that complies with the traceability requirements should contain the following minimum information (along with additional descriptions if necessary):

- the model equation [12],
- all input quantities (in the form of symbols) which were included in the uncertainty study,
- the (estimated) value of each input quantity [12],
- the associated standard uncertainty for each input quantity [12],
- details of correlations [12] and also covariances where applicable,
- the applied probability density function [12] *(e.g. normal distribution, rectangular distribution)*,
- the degrees of freedom [12] *(according to [GUM 4.2.6] always required for type A evaluation)*
- type of measurement uncertainty determination [12] *(type A or type B evaluation)*,
- the sensitivity coefficients,
- the uncertainty contributions to the output quantity,
- the value of output quantity,
- the combined standard uncertainty of the output quantity,
- the coverage factor [12] .

The form sheet shown in appendix I conforms to [VIM] and also contains further information.

4.8.2 Pareto chart and analysis of measurement uncertainty components

The Pareto chart is a graphical illustration of the Pareto principle according to which most consequences of a problem (typically around 80%) are frequently attributable to only a small number of causes (typically around 20%) [EQAT]. It is therefore advisable to identify these causes. In the case of measurement uncertainties, the Pareto chart is used to filter the largest uncertainty contributions out of the input quantities.

NOTE: The measurement uncertainty of a measuring system often can be significantly reduced by analyzing the component with the largest contribution according to Pareto and optimizing that component in order to reduce the uncertainty.

Examples: See appendix J, diagrams on pages 80, 89, 91, 102, 116 and 119.

[12] according to [VIM, 2.33, comment]

5 Approach according to ISO 22514-7 [13]

Chapters 1 and 2.5 explain that the **measurement uncertainty** provides an assertion about the range where the true value can be expected that is associated with a measured value. However, unlike **measuring process capability**, it makes no assertion as to whether measurement errors and dispersions of measured values are compatible with the tolerance zone of a characteristic (cf. chapter 2.5).

Whether both or only one of the two parameters (statistics) are required to ensure that defined requirements are met, usually can be decided based on the following criteria:

- Where measuring tasks change frequently (e.g. in development and testing departments), it is preferable to determine measurement uncertainties.
- Where a sufficiently large number of similar measurements of a specific characteristic are made repeatedly (e.g. in production), it is preferable to determine measuring process capabilities.
- If conformity statements are required according to [ISO 14253], it is essential to determine measurement uncertainties instead of or in addition to the proof of capabilities.

Capability and performance evaluations of production processes are based on measurement results. Substantiated assertions therefore require adequate consideration of the uncertainty to be allocated to the measuring process [14]. The procedures according to [AIAG MSA] and [Booklet 10] **globally** include all components of measurement uncertainty that are relevant to the measuring process into the evaluation results, since these uncertainties are already contained in the measurement results.

In contrast to this, [ISO 22514-7] provides a practice-oriented approach for the determination of measurement uncertainties based on [GUM] and the evaluation of the capability (suitability [15]) of measuring systems and measurement processes based on the determined **individual components** of the measurement uncertainty.

Initially the capability of the measuring system (MS) is determined and evaluated by means of the parameters Q_{MS} and C_{MS} with defined limit values.

Only after meeting these criteria the capability of the measuring process (MP) is determined and evaluated by means the parameters Q_{MP} and C_{MP} with defined limit values.

[13] The approach according to [VDA-5] corresponds to the approach according to [ISO 22514-7]
[14] In accordance with [ISO 22514-7], chap. “Introduction”
[15] Unlike the ISO standard, the German version of the VDA volume uses a German term which translates to English “suitability” or “appropriateness”. To ensure that the different language versions are unambiguous, the term “capability” is used throughout this guide including in the German version.

5.1 Procedure according to ISO 22514-7

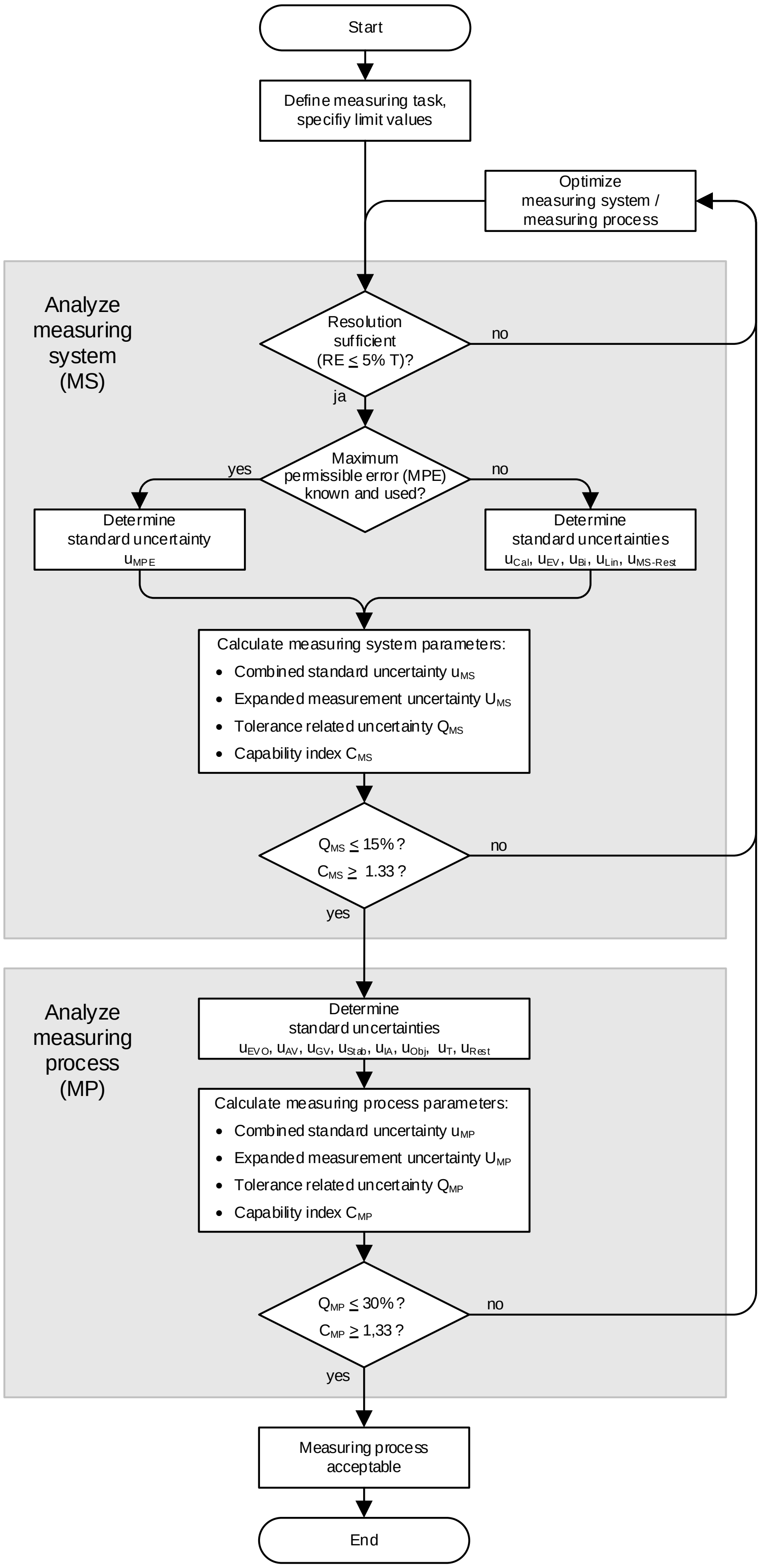


Figure 8: Procedure according to [ISO 22514-7] and limit values as recommended by the standard

5.2 Model equation

Model equations are not explicitly formulated in [ISO 22514-7]. However, the approach can be described according to [GUM] by means of the model equations

$$y_{MS} = y' + \delta x_{CAL} + \delta x_{EV(MS)} + \delta x_{BI} + \delta x_{LIN} + \delta x_{REST(MS)} \quad (5.1)$$

for the **measuring system** and

$$y_{MP} = y_{MS} + \left(\delta x_{EV(MP)} - \delta x_{EV(MS)}\right) + \delta x_{AV} + \delta x_{IA} + \delta x_{OBJ} + \delta x_{GV} + \delta x_{STAB} + \delta x_{\vartheta} + \delta x_{REST(MP)} \quad (5.2)$$

the **measuring process**. The equations represent a standardized specification based on an additive model (cf. chapter 4.3.1) with the following components:

y'	(Uncorrected) indication for the measurement results y_{MS} of the measuring system or y_{MP} of the measuring process,
δx_{CAL}	Deviation due to finite precision of calibration,
$\delta x_{EV(MS)}$	Deviation due to finite repeatability of the measuring system,
δx_{BI}	Systematic measurement error,
δx_{LIN}	Linearity error,
$\delta x_{REST(MS)}$	Deviation due to other influences attributable to the measuring system,
$\delta x_{EV(MP)}$	Deviation due to finite repeatability of the measuring process,
δx_{AV}	Deviation due to operator influence,
δx_{OBJ}	Deviation due to inhomogeneity of the measuring object, e.g. form deviations (if relevant),
δx_{IA}	Deviation due to interactions between input quantities,
δx_{STAB}	Deviation due to temporal instability of the measuring process,
δx_{ϑ}	Deviation due to temperature differences,
δx_{GV}	Deviation between different, technically comparable measuring systems (if relevant),
$\delta x_{REST(MP)}$	Deviation due to other influences attributable to the measuring process.

NOTE 1: The expected value of the deviation δx_i from the conventional value x_i of the input quantity i is 0. This applies to all input quantities i.

NOTE 2: The repeatability of the measuring system is one of several components that determines and also limits the repeatability of the measuring process in case all other components have no significant effect on the measuring process. Therefore, deviations of the measuring process caused by finite repeatability cannot be less than the corresponding deviations of the measuring system, so that the term $\delta x_{EV(MP)} - \delta x_{EV(MS)}$ cannot be negative.

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5.3 Uncertainties of the measurement system

The standard uncertainties $u(\delta x_i) = u_i$ of the input quantities i are determined as follows:

Uncertainty component	Symbol	Source, calculation
Calibration uncertainty *(Type B evaluation)*	$u(\delta x_{CAL}) = u_{CAL}$	Calibration certificate of the standards or manufacturer's data sheet: • If the expanded measurement uncertainty U_{CAL} is specified with the confidence level $(1-\alpha)\cdot 100\%$ it is divided by the corresponding coverage factor k_p: $u_{CAL} = \frac{U_{CAL}}{k_p}$ • If the confidence level is not specified, $k_p = 2$ is assumed. • Data that is not specified in more detail is adopted unchanged as standard uncertainty u_{CAL} (i.e. $k_p = 1$).
Resolution *(Type B evaluation)*	u_{RE}	Resolution RE taken from the manufacturer data sheet or estimated from readings: $u_{RE} = \frac{1}{\sqrt{3}} \frac{RE}{2}$ (Rectangular distribution)
Repeatability at the standard *(Type A evaluation, MSA study of type-1 or -4)*	u_{EVR}	$m \geq 30$ repeated measurements, calculation of the standard deviation s and the standard uncertainty (see [GUM], chap. 4.2.3; [Booklet 10], type-1 study) [16]: $u_{EVR} = \sqrt{\frac{1}{m-1} \cdot \sum_{k=1}^{m} (x_k - \bar{x})^2}$ Multiple standards: A total of $m \geq 30$ repeated measurements evenly distributed over all standards; common alternatives: • Determination of u_{EVR} for each standard (multiple type-1 study), determination of the maximum value of all u_{EVR} (see [VDA-5]); • Linear regression and estimation of u_{EVR} from the residual dispersion s of the measurement deviation around the regression line (see [Booklet 10], appendix E.1; [AIAG MSA]); • Determination of u_{EVR} and u_{LIN} by means of ANOVA [17].
	$u(\delta x_{EV(MS)})$ $= u_{EV(MS)}$	$u_{EV(MS)} = MAX(u_{RE}, u_{EVR})$
Systematic measurement error *(Type A evaluation, MSA study of type-1 or -4)*	$u(\delta x_{BI}) = u_{BI}$	$u_{BI} = \frac{\bar{x} - x_m}{\sqrt{3}}$ (Rectangular distribution) [18] $\bar{x}$ – Mean value of the measured values x_m – Reference value of the standard Multiple standards: Determination of u_{BI} for each standard (multiple type-1 study), determination of the maximum value of all u_{BI} (see [VDA-5]).

[16] [ISO 22514-7] provides no consideration of whether the smaller mean value dispersion can be used

[17] Analysis of Variances, abbreviated to ANOVA; mathematical method for decomposing variances into individual components

[18] Formula is applicable if systematic and random measurement errors are not distinguishable [ISO 22514-7]

Uncertainty component	Symbol	Source, calculation
Linearity error	$u(\delta x_{LIN}) = u_{LIN}$	• Ad-hoc assumption $u_{LIN} = 0$; • Calculation based on available limit values and adoption of a uniform distribution, e.g. $a = (a_+ - a_-)/2$ (cf. chap. 4.4.2.2): $u_{LIN} = \frac{a}{\sqrt{3}}$ (Rectangular distribution) • Experimental determination (see [Booklet 10], type-4 study, appendix E.1; [AIAG MSA], page 96 - 101); • Calibration certificate; • Determination u_{EVR} and u_{LIN} by means of ANOVA[17].
Residual deviations of the measuring system	$u(\delta x_{REST(MS)}) = u_{REST(MS)}$	If presumed or available: Determination based on tests (type A evaluation), data sheets, manufacturer's specifications, literature, etc. (type B evaluation)

Table 3: Uncertainty contributions of the measuring system according to [ISO 22514-7]

The combined standard uncertainty of the measuring system is calculated as

$$u_{MS} = \sqrt{u_{CAL}^2 + u_{EV(MS)}^2 + u_{BI}^2 + u_{LIN}^2 + u_{REST(MS)}^2} \tag{5.3}$$

(see chapter 4.5) and the expanded measurement uncertainty of the measuring system as

$$U_{MS} = k_p \cdot u_{MS} \tag{5.4}$$

(see chapter 4.6 and appendix D). A tabular uncertainty budget is not explicitly required by [ISO 22514-7].

5.4 Evaluation of the measuring system

For evaluating the capability of the measuring system the standard recommends the following parameters and limits:

$$Q_{MS} = \frac{2 \cdot U_{MS}}{T} \cdot 100\% \leq 15\% \tag{5.5}$$

$$C_{MS} = \frac{0{,}3 \cdot T}{6 \cdot u_{MS}} \geq 1{,}33 \tag{5.6}$$

NOTE 1: The following relationship exists between these two parameters

$$Q_{MS} = \frac{10\%}{C_{MS}} \cdot k_p$$

In the case $k_p > 2$ the criterion $Q_{MS} < 15\%$ represents the higher requirement for the measuring system, in case $k_p < 2$ the criterion $C_{MS} > 1.33$.

NOTE 2: The index C_{MS} must not be confused with the index C_g of a type-1 study [Booklet 10], since the standard uncertainty u_{MS} and the standard deviation s of a type-1 study are not equivalent in general. Equivalence assumes that the uncertainty contribution u_{EVR} (repeatability at the standard) is the only significant uncertainty component. This can be verified, for example by means of an uncertainty budget. However, even in this case the indexes are not comparable, since the use of the factor 0.3 instead of 0.2 means a reduction of the requirements according to [Booklet 10] and [CDQ0402] to 2/3, i.e. from 1.33 to 0.89.

Unless capability is achieved, the measuring system should be optimized before the measuring process is evaluated.

5.5 Uncertainties of the measuring process

Uncertainty component	**Symbol**	**Source, calculation**
Repeatability at the measuring object	u_{EVO}	Minimum requirements: • ≥ 30 data (sample size) from • ≥ 2 repeated measurements, • ≥ 5 measuring objects [ISO 22514-7] or ≥ 3 measuring objects [VDA-5], • ≥ 2 operators (if relevant), • ≥ 2 measuring devices (if relevant); Determination by means of ANOVA (see [Booklet 10], EV from a type-2 or type-3 study)
	$u(\delta x_{EV(MP)}) = u_{EV(MP)}$	$u_{EV(MP)} = MAX(u_{RE}, u_{EVR}, u_{EVO})$
Operator comparison	$u(\delta x_{AV}) = u_{AV}$	Minimum requirements: see u_{EVO}; Determination by means of ANOVA (see [Booklet 10], AV from a type-2 study)
Inhomogeneity of the individual measuring object	$u(\delta x_{OBJ}) = u_{OBJ}$	$u_{OBJ} = \frac{a_{OBJ}}{\sqrt{3}}$ (Rectangular distribution) Determination of the maximum deviation a_{OBJ} (e.g. shape): • Drawing (maximum permissible deviation) • Control chart (actual deviation) • Experiment (actual deviation) • Data sheet, manufacturer's specifications (estimate)
Interactions	$u(\delta x_{IA}) = u_{IA}$	$u_{IA} = \sqrt{\sum_{j=1}^{j_{max}} u_{IA\,j}^2}$ Determination of individual interactions $u_{IA\,j}$ by means of ANOVA (see [Booklet 10], type-2 study, IA operator - measuring object)
Instability of the measuring process over time	$u(\delta x_{STAB}) = u_{STAB}$	Minimum requirements: see u_{EVO}; Determination by means of ANOVA (see [Booklet 10], type 2 / 3 study)
Temperature	$u(\delta x_{\vartheta}) = u_{\vartheta}$	Possible determination of uncertainty from temperature differences in case of mechanical / geometric characteristics: $u_{\vartheta} = \sqrt{u_{TD}^2 + u_{TA}^2}$ • Temperature difference (according to ISO/TR 14523-2): $u_{TD} = \frac{\Delta\vartheta \cdot \alpha \cdot l}{\sqrt{3}}$ (Rectangular distribution) $\Delta\vartheta$ – Temperature change in K, α – Coefficient of expansion, l – Result of the length measurement. • Thermal expansion (according to ISO/TR 15530-3): $u_{TA} = \lvert \vartheta - 20°C \rvert \cdot u_{\alpha} \cdot l$ ϑ – Mean temperature in °C during the measurement, u_{α} – Standard uncertainty of the coefficients of expansion (e.g. from tables, data sheets or technical literature).

Uncertainty component	Symbol	Source, calculation
Comparability of different measurement systems	$u(\delta x_{GV}) = u_{GV}$	Relevant in case of more than one measuring system; consideration of the minimum and maximum values of individual and mean values measured for each reference part on the different measuring systems
Residual deviations of the measuring process	$u(\delta x_{REST(MP)}) = u_{REST(MP)}$	If presumed or available: Determination based on tests (type A evaluation), data sheets, manufacturer's specifications, literature, etc. (type B evaluation)

Table 4: Uncertainty contributions of the measuring process according to [ISO 22514-7]

The combined standard uncertainty u_{MP} of the measuring process is calculated as [19]

$$u_{MP} = \sqrt{u_{MS}^2 + \left(u_{EV(MP)}^2 - u_{EV(MS)}^2\right) + u_{AV}^2 + u_{OBJ}^2 + u_{IA}^2 + u_{STAB}^2 + u_{\vartheta}^2 + u_{GV}^2 + u_{REST(MP)}^2} \quad (5.7)$$

(see chapter 4.5) and the expanded measurement uncertainty of the measuring process as

$$U_{MP} = k_p \cdot u_{MP} \quad (5.8)$$

(see chapter 4.6 and appendix D). A tabular measurement uncertainty analysis is not explicitly required by [ISO 22514-7].

5.6 Evaluation of the measurement process

For evaluating the capability of the measuring process the standard recommends the following parameters and limits:

$$Q_{MP} = \frac{2 \cdot U_{MP}}{T} \cdot 100\% \leq 30\% \quad (5.9)$$

$$C_{MP} = \frac{0{,}3 \cdot T}{3 \cdot u_{MP}} \geq 1{,}33 \quad (5.10)$$

NOTE 1: The following relationship exists between these two parameters

$$Q_{MP} = \frac{20\%}{C_{MP}} \cdot k_p$$

In the case $k_p > 2$, the criterion $Q_{MS} < 30\%$ represents the higher requirement for the measuring process, in case $k_p < 2$ the criterion $C_{MP} > 1.33$.

NOTE 2: The index C_{MP} must not be confused with the index C_g of a type-1 study [Booklet 10], since the standard uncertainty u_{MP} and the standard deviation s of a type-1 study are not equivalent in general. Equivalence assumes that the uncertainty contribution u_{EVR} (repeatability at the standard) is the only significant uncertainty component. This can be verified, e.g. by means of a measurement uncertainty analysis. However, even in this case the indexes are not comparable, since the use of the factors 0.3 and 3 instead of 0.2 and 6 means a reduction of the requirements according to [Booklet 10] and [CDQ0402] to 1/3, i.e. from 1.33 to 0.44.

NOTE 3: With $k_p = 3$ the defining equation for Q_{MP} is formally transferred into the defining equation for %GRR. However, comparability with %GRR according to a type-2 study [Booklet 10] requires that u_{EVO} (repeatability at the test object), u_{AV} (operator comparison) and u_{IA} (interactions) are the only contributions to uncertainty which are verified as significant.

Unless capability is achieved, the entire process must be optimized.

[19] Only the <u>difference</u> $u_{EV(MP)}^2 - u_{EV(MS)}^2$ must be considered, since the fraction $u_{EV(MS)}^2$ is already included in u_{MS}^2. Mathematically, this eliminates the term $u_{EV(MS)}^2$ in u_{MS}^2 and replaces it with $u_{EV(MP)}^2$.

5.7 Maximum permissible error (MPE)

For the evaluation of the measuring system (MS) the concept of “maximum permissible error” (MPE) can be used as an alternative to the determination of measurement uncertainties according to [GUM].

The calibration of the measuring system or its components ensures that the equipment meets the requirements of defined metrological properties. This can be documented by specifying one or more MPE parameters.

MPE can be particularly useful when several similar, but physically different, measuring systems are used for a measuring process. If only one measuring system is used, the experimental method according to [GUM] is usually more advantageous, since it provides lower measurement uncertainties.

Using MPE to evaluate the measuring system and the measuring process can be described by following model equation:

$$y_{MS} = y' + \delta x_{MPE} \quad (5.11)$$

$$y_{MP} = y_{MS} + \delta x_{AV} + \delta x_{OBJ} + \delta x_{IA} + \delta x_{GV} + \delta x_{STAB} + \delta x_{\vartheta} + \delta x_{REST(MP)} \quad (5.12)$$

with

δx_{MPE} deviation less than or at most equal to the maximum permissible error MPE.

Uncertainty component	**Symbol**	**Source, calculation**
Maximum permissible error	$u(\delta x_{MPE}) = u_{MPE}$	$u_{MPE} = \frac{MPE}{\sqrt{3}}$ (Rectangular distribution) In the case of several MPE values that can affect the measurement result: $u_{MPE} = \sqrt{\frac{MPE_1^2}{3} + \frac{MPE_2^2}{3} + \ldots + \frac{MPE_n^2}{3}}$ MPE values are taken e.g. from the calibration documents

Table 5: Contribution of the maximum permissible error to uncertainty

The remaining uncertainty components are determined according to Table 4. The combined standard uncertainty and the expanded measurement uncertainty of the measuring process are calculated as

$$u_{MP} = \sqrt{u_{MPE}^2 + u_{AV}^2 + u_{OBJ}^2 + u_{IA}^2 + u_{STAB}^2 + u_{\vartheta}^2 + u_{GV}^2 + u_{REST(MP)}^2} \quad (5.13)$$

$$U_{MP} = k_p \cdot u_{MP} . \quad (5.14)$$

NOTE: [ISO 22514-7] does not include any information about how these equations take account of the case “u_{EVO} greater than u_{EVR} and u_{RE}”.

6 Measurement uncertainty based on the procedures according to booklet 10 and ISO 22514-7

Capability studies according to [Booklet 10] require carrying out several specific investigations (studies of type 1 to 5). These procedures take into account influences on the measuring result such as the measuring system, operators, measuring objects, measurement strategy, environmental conditions and stability over time. Thus, most uncertainty components according to [ISO 22514-7] are already included in the measurement data. This data can be used to determine a value for the measurement uncertainty U so that the requirements of various standards and guidelines for identifying and taking account of the uncertainty of measurement results (cf. chap. 1) are met without any additional investigation effort. If these data are not available or merely partially available, the following explanations do not apply and the procedures according to chap. 4 or, where appropriate, chap. 5 have to be used.

The uncertainty components according to [ISO 22514-7] are primarily attributed to the following data sources:

Symbol	**Uncertainty component** according to chap. 5.2 (ISO 22514-7)	**Included in information or data source**
u_{CAL}	Deviation δx_{CAL} due to finite precision of calibration	**Calibration certificate** of the standard or reference part used
$u_{EV(MS)}$	Deviation $\delta x_{EV(MS)}$ due to finite repeatability of the measuring system	**Type-5 study:** Dispersion of the measuring system with a standard or a reference part
u_{BI}	Systematic measurement error δx_{BI}	**Type-5 study:** Mean deviation of the measured values from the reference value of the standard or series part
u_{LIN}	Linearity error δx_{LIN}	*If relevant according to chap. 5.3, Table 3*
$u_{REST(MS)}$	Deviation $\delta x_{REST(MS)}$ due to other influences attributable to the measuring system	**Type-5 study:** All other influences not mentioned above, that are **not** caused by series parts
$u_{EV(MP)}$	Deviation $\delta x_{EV(MP)}$ due to finite repeatability of the measuring process	**Type-1 and type-2/3 studies (difference):** Increase in measuring system dispersion due to series parts
u_{AV}	Deviation δx_{AV} due to operator influence	**Type-5 study:** Dispersion as a result of different operators
u_{OBJ}	Deviation δx_{OBJ} due to inhomogeneity of the **individual** measuring object, e.g. caused by shape variation, surface quality or material properties	*If relevant according to chap. 5.5, Table 4*
$u_{IA(1)}$	Deviation $\delta x_{IA(1)}$ due to interactions between input quantities	**Type-5 study:** Interactions that are not caused by series parts
$u_{IA(2)}$	Deviation $\delta x_{IA(2)}$ due to interactions between input quantities	**Type-2 study:** Interactions between operators and series parts
u_{STAB}	Deviation δx_{STAB} due to instability of the measuring process over time *"Reproducibility over time" [ISO 22514-7, pp. 21]*	**Type-5 study:** Dispersion as a result of deviations from the long term stability of the measuring process
u_{ϑ}	Deviation δx_{ϑ} due to temperature differences	**Type-5 study:** Influence of temperature changes and settings that deviate from the nominal value
u_{GV}	Deviation δx_{GV} between different, but technically comparable measuring systems	*If relevant according to chap. 5.5, Table 4*
$u_{REST(MP)}$	Deviation $\delta x_{REST(MP)}$ due to other influences attributable to the measuring process	*If relevant according to chap. 5.5, Table 4*

The individual uncertainty components are defined according to the model equation

$$y = y' + \delta x_{CAL} + \delta x_{BI} + \delta x_{PRO} + \delta x_{PAR} + \delta x_{EXT} \tag{6.1}$$

and applied or combined as follows:

- Standard uncertainty of the calibration of the standard or reference part used *(**cal**ibration)*:

$$u_{CAL} \tag{6.2}$$

- Standard uncertainty due to uncorrected, systematic measurement errors *(**bi**as)*:

$$u_{BI} \tag{6.3}$$

- Standard uncertainty of the measurement procedure *(**pro**cedure)*; random and uncorrected deviations under intermediate precision conditions caused by the measuring system, the standard, the operator, time and environment:

$$u_{PRO} = \sqrt{u_{EV(MS)}^2 + u_{REST(MS)}^2 + u_{AV}^2 + u_{IA(1)}^2 + u_{STAB}^2 + u_{\vartheta}^2} \approx \sqrt{\frac{1}{m-1} \cdot \sum_{k=1}^{m} (x_k - \bar{x})^2} \tag{6.4}$$

- Standard uncertainty due to series parts *(**par**ts)*; influence of the measurement strategy and the measuring object during measurements on series parts:

$$u_{PAR} = \sqrt{u_{EV(MP)}^2 - u_{EV(MS)}^2} \approx \sqrt{EV^2 - s^2} \tag{6.5}$$

- Additional standard uncertainty due to other influences *(**ext**ra)*; provided that one or more individual components are relevant (if applicable see chap. 5.3, Table 3, and chap. 5.5, Table 4):

$$u_{EXT} = \sqrt{u_{LIN}^2 + u_{OBJ}^2 + u_{IA(2)}^2 + u_{GV}^2 + u_{REST(MP)}^2} \tag{6.6}$$

6.1 Determining uncertainty components

6.1.1 Standard uncertainty u_{CAL} of the standard calibration

The value of the expanded measurement uncertainty U_{CAL} must be taken from the calibration certificate of the standard or reference part and divided by the coverage factor k_p (k_p = 2 at confidence level 95.45%):

$$u_{CAL} = \frac{U_{CAL}}{k_p} \tag{6.7}$$

6.1.2 Standard uncertainty u_{BI} due to a systematic measurement error

The difference between the mean value $\bar{x}$ of the measured values of the relevant stability charts and the conventional value x_m of the standard or reference part according to the calibration certificate must be taken into account as a standard uncertainty according to appendix F:

$$u_{BI} = x_m - \bar{x} \tag{6.8}$$

*NOTE 1: In contrast to [ISO 22514-7] and according to [GUM], this difference is applied **unmodified** as standard uncertainty u_{BI}. Alternatively, a corresponding correction can be made and the uncertainty contribution u_{BI} can be omitted.*

NOTE 2: The uncertainty of this standard uncertainty (or correction) is contained in the dispersion of the measured values. Thus, it is already included via u_{PRO} in the measurement uncertainty of the measuring process and does not need to be considered separately.

6.1.3 Standard uncertainty u_{PRO} of the measurement procedure

The individual values documented in the stability chart (type-5 study) represent the dispersion of the measuring process under varying external conditions (e.g. variations of temperature or measuring force or changes of operators). At least m = 25 measurements should be available.

$$u_{PRO} = \sqrt{\frac{1}{m-1} \cdot \sum_{k=1}^{m} \left(x_k - \bar{x}\right)^2} \tag{6.9}$$

NOTE 1: Since the expanded measurement uncertainty U to be determined here shall refer to individual measurements, u_{PRO} corresponds to the standard deviation of all individual values.

*NOTE 2: A type-5 study is performed using a standard just as used with a type-1 study or a reference part (stability part). Therefore, uncertainty components resulting from series parts are **not** included in the measurement results and must be considered separately.*

6.1.4 Standard uncertainty u_{PAR} of the measuring object

Unlike a type-1 study, an additional uncertainty component u_{PAR} is usually effective in case of measurements on series parts (e.g. caused by shape deviations). That is why EV from a type-2 or a type-3 study usually is larger than s from a type-1 study. This difference is significant if the condition

$$EV^2 > 2\,s^2 \tag{6.10}$$

is fulfilled. Only then, u_{PAR} must be taken into account:

$$u_{PAR} = \sqrt{EV^2 - s^2} \tag{6.11}$$

NOTE: The criterion $EV^2/s^2 > 2$ is based on an F-test with a confidence level of 95% and approximately 20 – 30 individual values for determining EV or s, respectively; the corresponding quantiles of the F-distribution are in the 1.85 to 2.15 value range.

6.1.5 Standard uncertainty u_{EXT} of other uncertainty components

If the influence of other uncertainty components (such as linearity, homogeneity, interactions or system differences) is evaluated or assumed to be relevant:

$$u_{EXT} = \sqrt{u_{LIN}^2 + u_{OBJ}^2 + u_{IA(2)}^2 + u_{GV}^2 + u_{REST(MP)}^2} \tag{6.12}$$

6.2 Combined standard uncertainty u_C

$$u_C = \sqrt{u_{CAL}^2 + u_{BI}^2 + u_{PRO}^2 + u_{PAR}^2 + u_{EXT}^2} \tag{6.13}$$

6.3 Expanded measurement uncertainty U

$$U = k_p \cdot u_C \tag{6.14}$$

The calculated measurement uncertainty U applies to an individual measurement and the period being considered (according to the stability chart). $k_p = 2$ applies to a confidence interval of 95.45%.

NOTE: The uncertainties u_C and U can be utilized for a capability evaluation of the measuring process according to [ISO 22514-7] (cf. chapter 6.5).

6.4 Complete measurement result y

$$y = y' \pm U \tag{6.15}$$

Application examples: See chapter 6.5 and appendix J.6

6.5 Example from booklet 10: Outer diameter of a shaft

Required data

- Calibration or test certificate providing reference value and calibration uncertainty of the standard;
- Results from type-1 and type-2 (or -3) studies;
- Stability chart with at least 25 sample results (type-5 study);
- Tolerance of the characteristic (in this case T = 0.06 mm = 60 µm).

NOTE: The data for this example were taken from the forms shown in [Booklet 10], chap. 4.

Standard uncertainty u_{CAL} of the calibration of the standard

The calibration certificate of the standard provides the reference value $x_m = 6.002\,\text{mm}$ and $U_{CAL} = 0.001\,\text{mm}$. The standard uncertainty u_{CAL} is calculated by dividing the uncertainty U_{CAL} by the coverage factor k_p (here $k_p = 2$):

$$u_{CAL} = \frac{U_{CAL}}{k_p} = \frac{0{,}001}{2}\,\text{mm} = 0{,}0005\,\text{mm} = 0{,}5\,\mu\text{m}$$

Standard uncertainty u_{BI} due to systematic measurement error

Reference value: $x_m = 6{,}002\,\text{mm} = 6002\,\mu\text{m}$ from the calibration certificate of the standard,

Mean value: $\bar{x} = 6{,}002\,\text{mm} = 6002\,\mu\text{m}$ from type-5 study.

$$u_{BI} = x_m - \bar{x} = 6002\,\mu\text{m} - 6002\,\mu\text{m} = 0\,\mu\text{m}$$

Standard uncertainty u_{PRO} of the measurement procedure

u_{PRO} is the standard deviation of all individual values in the stability chart:

$u_{PRO} = 0{,}0013\,\text{mm} = 1{,}3\,\mu\text{m}$ from type-5 study.

Standard uncertainty u_{PAR} by measurements on series parts

In addition to s from a type-1 study, EV from both type-2 and type-3 studies is available in this example:

$s = 0{,}00100\,\text{mm} = 1{,}00\,\mu\text{m}$ from type-1 study,

$EV = 0{,}00153\,\text{mm} = 1{,}53\,\mu\text{m}$ from type-2 study,

$EV = 0{,}00147\,\text{mm} = 1{,}47\,\mu\text{m}$ from type-3 study.

The larger one of the two standard deviations EV is used (type-2 study):

$EV^2 = 2{,}34\,\mu\text{m}^2 > 2 \cdot s^2 = 2{,}00\,\mu\text{m}^2$ i.e. the difference is significant.

Accordingly, Eq. (6.11) must be taken into account:

$$u_{PAR} = \sqrt{EV^2 - s^2} = \sqrt{1{,}53^2 - 1{,}00^2}\,\mu\text{m} \approx 1{,}2\,\mu\text{m}$$

Standard uncertainty u_{EXT} due to other uncertainty components

Interactions operators – parts are insignificant, other components are considered non-relevant.

Combined standard uncertainty u_C

$$u_C = \sqrt{u_{CAL}^2 + u_{BI}^2 + u_{PRO}^2 + u_{PAR}^2} = \sqrt{0{,}5^2 + 0{,}0^2 + 1{,}3^2 + 1{,}2^2}\,\mu\text{m} = \sqrt{3{,}4}\,\mu\text{m} \approx 1{,}8\,\mu\text{m}$$

Expanded measurement uncertainty U for the considered period

$$U = k_p \cdot u_C = 2 \cdot 1{,}8\,\mu\text{m} = 3{,}6\,\mu\text{m}$$

Classification

The capability requirements recommended by [ISO 22514-7] are met:

$$Q_{MP} = \frac{2 \cdot U_{MP}}{T} \cdot 100\% = \frac{2 \cdot U}{T} \cdot 100\% = \frac{2 \cdot 3{,}6\,\mu\text{m}}{60\,\mu\text{m}} \cdot 100\% = 12\% \le 30\% \qquad (6.16)$$

$$C_{MP} = \frac{0{,}3 \cdot T}{3 \cdot u_{MP}} = \frac{0{,}3 \cdot T}{3 \cdot u_C} = \frac{0{,}3 \cdot 60\,\mu\text{m}}{3 \cdot 1{,}8\,\mu\text{m}} = \frac{6}{1{,}8} = 3{,}33 \ge 1{,}33 \qquad (6.17)$$

The requirement according to the “golden rule of metrology” is also met:

$$\frac{U}{T} = \frac{3{,}6\,\mu\text{m}}{60\,\mu\text{m}} = 0{,}06 \le 0{,}1 \qquad (6.18)$$

Appendix

A Examples of input quantities and influences

The following list – which is not exhaustive – contains typical examples from different categories which can serve as a guide in determining input quantities.

Environmental influences

- Temperature: Absolute temperature, spatial and temporal gradient
- Vibrations
- Noise
- Humidity
- Contamination
- Lighting
- Atmospheric pressure
- Air composition
- Air draft
- Gravity
- Electrical interference fields
- Power supply variations
- Pressure variations in the compressed air supply
- Heat radiation
- Influence of the measuring object
- Thermal equilibrium of the measuring instrument

Standards and material measures

- Stability
- Quality of the reference
- Physical principle of the reference: analog, optically digital, magnetically digital, toothed rack, interferometry
- Uncertainty of calibration
- Resolution of the standard instrument
- Thermal coefficient of expansion

Measuring system

- Resolution
- Output system
- Mechanical or electrical boost
- Wavelength error
- Stability of the zero point
- Stability of the measuring force, absolute force
- Hysteresis
- Accuracy of mechanical guidance
- Probe system
- Stiffness, elasticity
- Reading head of the measuring system
- Thermal expansion
- Parallax
- Time since last calibration
- Sensitivity characteristics
- Interpolation system
- Resolution of interpolation
- Digitizing

Measuring the measuring object

- Cosine and sine errors
- Violation of the Abbe principle
- Temperature sensitivity
- Stiffness and elasticity
- Probe tip radius
- Flattening of the probe tip
- Stiffness of the stylus
- Optical aperture
- Influence of clamping device on the measuring object
- Thermal compensation

Data processing

- Rounding rules
- Algorithms
- Application of algorithms
- Number of significant places used in the calculation
- Sample
- Filtering
- Certification of algorithms
- Interpolation and extrapolation
- Outlier handling

Human influence

- Experience
- Training
- Physical and mental condition
- Expertise
- Honesty
- Interest in the task
- Diligence

Properties of the measuring object

- Surface roughness
- Form deviation
- Elastic modulus (E-modulus)
- Stability beyond the elastic modulus
- Thermal coefficient of expansion
- Electrical conductivity
- Weight
- Dimensions
- Surface
- Magnetism
- Hygroscopic property
- Aging
- Cleanliness
- Temperature
- Internal stress
- Creep characteristics
- Object deformation during clamping on the measuring instrument

Definition of characteristics

- Date
- Reference system
- Degrees of freedom
- Assessment methods (e.g. surface texture, ISO 4288)
- Distance
- Angle
- Toleranced characteristics

Measurement methods

- Course of action
- Number of measurements
- Sequence of measurements
- Duration of the measurement
- Choice of measuring principle
- Alignment
- Choice of reference, reference object
- Alignment of the probe system
- Choice of equipment
- Choice of operators
- Number of operators
- Strategy
- Measuring object fastening
- Number of measuring points
- Probe head system
- Drift behavior

B Calculation of sensitivity coefficients

The combined standard uncertainty for any (linear) model

$$y = f(x_1, x_2, \ldots, x_n) \tag{B.1}$$

for n **uncorrelated** input quantities x_i with $1 \le i \le n$ is calculated as

$$u_c(y) = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial y}{\partial x_i} \cdot u(x_i) \right)^2} = \sqrt{\sum_{i=1}^{n} \left(c_i \cdot u(x_i) \right)^2} \tag{B.2}$$

with the sensitivity coefficients

$$c_i = \frac{\partial y}{\partial x_i} \tag{B.3}$$

B.1 Additive model

$$y = \sum_{i=1}^{n} x_i = x_1 + x_2 + \ldots + x_n \tag{B.4}$$

The sensitivity coefficient c_1 is calculated as

$$c_1 = \frac{\partial y}{\partial x_1} = \frac{\partial \sum_{i=1}^{n} x_i}{\partial x_1} = \frac{\partial (x_1 + x_2 + \ldots + x_n)}{\partial x_1} = \frac{\partial x_1}{\partial x_1} + \frac{\partial x_2}{\partial x_1} + \ldots + \frac{\partial x_n}{\partial x_1} = 1 + 0 + \ldots + 0 = 1$$

or any sensitivity coefficient c_k with $1 \le k \le n$ as

$$c_k = \frac{\partial y}{\partial x_k} = \frac{\partial \sum_{i=1}^{n} x_i}{\partial x_k} = \frac{\partial (x_1 + x_2 + \ldots + x_k + \ldots + x_n)}{\partial x_k} = \frac{\partial x_k}{\partial x_k} = 1 \tag{B.5}$$

The combined standard uncertainty is then calculated as

$$u_c(y) = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial y}{\partial x_i} \cdot u(x_i) \right)^2} = \sqrt{\sum_{i=1}^{n} (1 \cdot u(x_i))^2} = \sqrt{\sum_{i=1}^{n} u^2(x_i)} = \sqrt{u^2(x_1) + u^2(x_2) + \ldots + u^2(x_n)} \tag{B.6}$$

B.2 Multiplicative model

$$y = \frac{\prod_{i=1}^{m} x_i}{\prod_{i=m+1}^{n} x_i} = \frac{x_1 \cdot x_2 \cdot \ldots \cdot x_m}{x_{m+1} \cdot x_{m+2} \cdot \ldots \cdot x_n} \tag{B.7}$$

A special case of this model equation is

$$y = \prod_{i=1}^{n} x_i = x_1 \cdot x_2 \cdot \ldots \cdot x_n \tag{B.8}$$

In this case the sensitivity coefficient c_1 is calculated as

$$c_1 = \frac{\partial y}{\partial x_1} = \frac{\partial \prod_{i=1}^{n} x_i}{\partial x_1} = \frac{\partial (x_1 \cdot x_2 \cdot \ldots \cdot x_n)}{\partial x_1} = 1 \cdot x_2 \cdot \ldots \cdot x_n = \frac{x_1 \cdot x_2 \cdot \ldots \cdot x_n}{x_1} = \frac{y}{x_1}$$

or any sensitivity coefficient c_k with $1 \le k \le n$ as

$$c_k = \frac{\partial y}{\partial x_k} = \frac{\partial \prod_{i=1}^{n} x_i}{\partial x_k} = \frac{\partial (x_1 \cdot x_2 \cdot \ldots \cdot x_k \cdot \ldots \cdot x_n)}{\partial x_k} = \frac{y}{x_k} \tag{B.9}$$

Another special case of this model equation is

$$y = \prod_{i=1}^{n} \frac{1}{x_i} = \frac{1}{x_1} \cdot \frac{1}{x_2} \cdot \ldots \cdot \frac{1}{x_n} \tag{B.10}$$

In this case the sensitivity coefficient c_1 is calculated as

$$c_1 = \frac{\partial y}{\partial x_1} = \frac{\partial \prod_{i=1}^{n} x_i}{\partial x_1} = \frac{\partial}{\partial x_1}\left(\frac{1}{x_1} \cdot \frac{1}{x_2} \cdot \ldots \cdot \frac{1}{x_n}\right) = \frac{-1}{x_1^2} \cdot \frac{1}{x_2} \cdot \ldots \cdot \frac{1}{x_n} = -\frac{1}{x_1}\left(\frac{1}{x_1} \cdot \frac{1}{x_2} \cdot \ldots \cdot \frac{1}{x_n}\right) = -\frac{y}{x_1}$$

or any sensitivity coefficient c_k with $1 \le k \le n$ as

$$c_k = \frac{\partial y}{\partial x_k} = \frac{\partial \prod_{i=1}^{n} x_i}{\partial x_k} = \frac{\partial}{\partial x_k}\left(\frac{1}{x_1} \cdot \frac{1}{x_2} \cdot \ldots \cdot \frac{1}{x_k} \cdot \ldots \cdot \frac{1}{x_n}\right) = -\frac{y}{x_k} \tag{B.11}$$

The sensitivity coefficients of both special cases differ only in terms of the sign. Because the sensitivity coefficients are squared for the calculation of the combined standard uncertainty, the sign is not relevant. Thus, for both special cases and the general case, the combined standard uncertainty is always calculated according to

$$u_c(y) = \sqrt{\sum_{i=1}^{n}\left(\frac{\partial y}{\partial x_i} \cdot u(x_i)\right)^2} = \sqrt{\sum_{i=1}^{n}\left(\frac{y}{x_i} \cdot u(x_i)\right)^2} = y \cdot \sqrt{\sum_{i=1}^{n}\left(\frac{u(x_i)}{x_i}\right)^2} = y \cdot \sqrt{\left(\frac{u(x_1)}{x_1}\right)^2 + \left(\frac{u(x_2)}{x_2}\right)^2 + \ldots + \left(\frac{u(x_n)}{x_n}\right)^2} \tag{B.12}$$

or by means of this equation divided by y:

$$\frac{u_c(y)}{y} = \sqrt{\sum_{i=1}^{n}\left(\frac{u(x_i)}{x_i}\right)^2} = \sqrt{\left(\frac{u(x_1)}{x_1}\right)^2 + \left(\frac{u(x_2)}{x_2}\right)^2 + \ldots + \left(\frac{u(x_n)}{x_n}\right)^2} \tag{B.13}$$

B.3 Linear function

$$y = \sum_{i=1}^{n} (a_i + b_i \cdot x_i) = (a_1 + b_1 \cdot x_1) + (a_2 + b_2 \cdot x_2) + \ldots + (a_n + b_n \cdot x_n) \tag{B.14}$$

The linear function according to Eq. (B.14) mathematically represents a combination of the additive model and the multiplicative model. The constants a_i and b_i are also subject to uncertainty since usually they are not precisely known. If, in the first step, only additive relations are considered, the following results according to Eq. (B.6):

$$u_c(y) = \sqrt{\sum_{i=1}^{n} u^2(a_i) + \sum_{i=1}^{n} u^2(b_i \cdot x_i)} \tag{B.15}$$

Then, in the second step, the multiplicative relations in the second summand is considered according to Eq. (B.12) so that

$$u_c(y) = \sqrt{\sum_{i=1}^{n} u^2(a_i) + \sum_{i=1}^{n}\left[\left(b_i \cdot x_i \frac{u(b_i)}{b_i}\right)^2 + \left(b_i \cdot x_i \frac{u(x_i)}{x_i}\right)^2\right]} = \sqrt{\sum_{i=1}^{n}\left(u^2(a_i) + x_i^2 \cdot u^2(b_i) + b_i^2 \cdot u^2(x_i)\right)}$$

or

$$u_c(y) = \sqrt{u^2(a_1) + x_1^2 u^2(b_1) + b_1^2 u^2(x_1) + \ldots + u^2(a_n) + x_n^2 u^2(b_n) + b_n^2 u^2(x_n)} \tag{B.16}$$

In the special case n = 1 (linear equation) the following applies:

$$y = a + b \cdot x \tag{B.17}$$

with the standard uncertainty

$$u_c(y) = \sqrt{u^2(a) + x^2 \cdot u^2(b) + b^2 \cdot u^2(x)} \tag{B.18}$$

C Correlated input quantities

NOTE 1: The consideration of correlations make higher demands on the user's physical and mathematical understanding.

NOTE 2: The applicability of the following observations presupposes that there is a linear relationship between the correlated quantities.

C.1 Uncertainties of input quantities

In case of a type A evaluation the correlation of the input quantities i and j can be verified by means of the **covariance** of the data sets x_{ik} and x_{jk}, each consisting of m measured values:

$$s(x_i, x_j) = \frac{1}{m-1} \cdot \sum_{k=1}^{m} (x_{ik} - \bar{x}_i) \cdot (x_{jk} - \bar{x}_j) \tag{C.1}$$

The covariance related to the product of both standard deviations $s(x_i)$ and $s(x_j)$ is referred to as **correlation coefficient**:

$$r(x_i, x_j) = \frac{s(x_i, x_j)}{s(x_i) \cdot s(x_j)} \tag{C.2}$$

The value of $r(x_i, x_j)$ is a measure for the strength of the correlation:

$r(x_i, x_j) = +1$	complete positive correlation (e.g. $x_j = +a \cdot x_i + b$),
$r(x_i, x_j) = 0$	no correlation,
$r(x_i, x_j) = -1$	complete negative correlation (e.g. $x_j = -a \cdot x_i + b$).

The covariances of the correlated mean values $\bar{x}_i$ and $\bar{x}_j$ are calculated as

$$u(\bar{x}_i, \bar{x}_j) = \frac{s(x_i, x_j)}{m} \tag{C.3}$$

NOTE 1: For the applicability of m > 1 the note 2 in chap. 4.4.1.1 must be considered.

In practice, the representation using correlation coefficients and standard uncertainties of the mean values is mostly preferred:

$$u(\bar{x}_i, \bar{x}_j) = r(x_i, x_j) \cdot u(\bar{x}_i) \cdot u(\bar{x}_j) \tag{C.4}$$

NOTE 2: The notation with horizontal bars on the x_i and x_j means that mean values are concerned. The relationships nevertheless apply in the same way if the x_i and/or x_j were not determined as mean values.

Based on these equations, it is easy to verify that the following relationships always apply for the statistical quantities defined above:

$s(x_i, x_j) = s(x_j, x_i)$	$s(x_i, x_i) = s^2(x_i)$	$s(x_j, x_j) = s^2(x_j)$
$r(x_i, x_j) = r(x_j, x_i)$	$r(x_i, x_i) = 1$	$r(x_j, x_j) = 1$
$u(x_i, x_j) = u(x_j, x_i)$	$u(x_i, x_i) = u^2(x_i)$	$u(x_j, x_j) = u^2(x_j)$

According to [GUM] the covariances $s(x_i, x_j)$ or $u(x_i, x_j)$ or the correlation coefficients $r(x_i, x_j)$ have to be specified in addition to the standard uncertainties $u(x_i)$ and $u(x_j)$ in case of correlated input quantities. They are usually represented as elements of matrices.

The diagonal elements of the **covariance matrix** are the squares of the standard deviations (i.e. the variances) of the input quantities; the non-diagonal elements are the covariances. Example of 3 input quantities x_1, x_2 and x_3:

$$\mathbf{s} = \begin{pmatrix} s^2(x_1) & s(x_1, x_2) & s(x_1, x_3) \\ s(x_2, x_1) & s^2(x_2) & s(x_2, x_3) \\ s(x_3, x_1) & s(x_3, x_2) & s^2(x_3) \end{pmatrix} \tag{C.5}$$

The diagonal elements of the **correlation coefficient matrix** are 1, the non-diagonal elements are the correlation coefficients. Example of 3 input quantities x_1, x_2 and x_3:

$$\mathbf{r} = \begin{pmatrix} 1 & r(x_1,x_2) & r(x_1,x_3) \\ r(x_2,x_1) & 1 & r(x_2,x_3) \\ r(x_3,x_1) & r(x_3,x_2) & 1 \end{pmatrix} \tag{C.6}$$

The **uncertainty matrix** is analogous to the covariance matrix of the individual values according to Eq. (C.5). The diagonal elements are the squares of the standard uncertainties of the mean values. Example of 3 input quantities x_1, x_2 and x_3:

$$\mathbf{u} = \begin{pmatrix} u^2(x_1) & u(x_1,x_2) & u(x_1,x_3) \\ u(x_2,x_1) & u^2(x_2) & u(x_2,x_3) \\ u(x_3,x_1) & u(x_3,x_2) & u^2(x_3) \end{pmatrix} \tag{C.7}$$

The following approximation can be used as a basis for the empirical determination. If the variation δx_i of an input quantity i with standard uncertainty $u(x_i)$ causes a variation δx_j of the correlated input quantity j with standard uncertainty $u(x_j)$, the following relationship applies approximately [GUM, C.3.6, note 3]:

$$r(x_i,x_j) \approx \frac{u(x_i)\cdot \delta x_j}{u(x_j)\cdot \delta x_i} \tag{C.8}$$

NOTE 3: It is important to note that $r(x_i, x_j) = r(x_j, x_i)$ exactly applies in the special case $u(x_i) / u(x_j) = |\delta x_i / \delta x_j|$ only. This special case must be met in good approximation, so that calculations of the combined standard uncertainty according to Eq. (C.9) provide acceptable results.

In case a type B evaluation is required for one or more input quantities, the covariances usually can be calculated only partially or not at all by means of Eq. (C.1). Instead, estimated values are used for the elements of the correlation coefficient matrix.

NOTE 4: If there is a positive (negative) correlation with $r > 0$ ($r < 0$), the correlation coefficient can be estimated with $r(x_i, x_j) = 0.5$ (-0.5) in case more detailed information is unavailable (see [EUROLAB, A.6.4]).

NOTE 5: If the magnitudes of the standard uncertainties of input quantities are very different, correlations are negligible under certain circumstances.

Correlation coefficients take values in the range $-1 \leq r(x_i, x_j) \leq +1$. Thus, the condition $|u(x_i, x_j)| \leq u(x_i) \cdot u(x_j)$ results from Eq. (C.4). If one of the two uncertainties $u(x_i)$ or $u(x_j)$ is small in relation to the other, the absolute value of covariance $|u(x_i, x_j)|$ is also small. Examples:

- *The standard uncertainties $u(x_1) = 0.80$ and $u(x_2) = 0.02$ have been determined. Even in the worst case of full correlation $|r(x_1, x_2)| = 1$, the absolute value of the covariance $|u(x_1, x_2)|$ cannot not take values greater than $u(x_1) \cdot u(x_2) = 0.80 \cdot 0.02 = 0.016$. Thus, the covariance cannot amount to more than 2.5% of the total variance $u^2(x_1) + u^2(x_2) = 0.80^2 + 0.02^2 \approx 0.64$. This may be neglected.*
- *In the case $u(x_2) = 0.90$, the covariance can rise to $u(x_1) \cdot u(x_2) = 0.80 \cdot 0.90 = 0.72$ in the worst case and represent approximately 50% of the total variance $u^2(x_1) + u^2(x_2) = 0.80^2 + 0.90^2 = 1.45$. This is not negligible.*

C.2 Calculating the combined standard uncertainty

According to [GUM, 5.2] the following calculation rule applies to the combined standard uncertainty:

$$u_c(y) = \sqrt{\sum_{i=1}^{n} c_i^2 \cdot u^2(x_i) + 2\cdot \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} c_i \cdot u(x_i)\cdot r(x_i,x_j)\cdot c_j \cdot u(x_j)} \tag{C.9}$$

with the sensitivity coefficients

$$c_i = \frac{\partial y}{\partial x_i} = \frac{\partial f(x_1,\ldots,x_n)}{\partial x_i} \quad \text{and} \quad c_j = \frac{\partial y}{\partial x_j} = \frac{\partial f(x_1,\ldots,x_n)}{\partial x_j} \quad .$$

This calculation rule represents a generalization of Eq. (4.21). Unlike Eq. (4.21), it applies to both correlated and uncorrelated input quantities.

NOTE 1: In the case of uncorrelated input quantities i and j, $r(x_i, x_j) = 0$ applies. Therefore, these input quantities do not make any contribution to the double sum in Eq. (C.9). If all input quantities are uncorrelated, the double sum disappears and Eq. (C.9) reduces to Eq. (4.21).

NOTE 2: If all input quantities are fully correlated, i.e. $r(x_i, x_j) = +1$ or $r(x_i, x_j) = -1$ applies to all i and j, the combined standard uncertainty results from a simple arithmetic addition of the standard uncertainties of the individual input quantities rather than an addition of the squared quantities [GUM, 5.2.2, NOTE 1]. In this case the uncertainties can compensate for each other. This effect can be easily verified by means of Eq. (C.12).

NOTE 3: Appendix B provides sensitivity coefficients for specific model equations.

C.3 Mathematical supplements

C.3.1 Covariances and standard uncertainties of mean values

Covariances of mean values can be described by means of Eq. (C.4) using the correlation coefficients and standard uncertainties of the mean values. For this purpose Eq. (C.2) is solved for $s(x_i, x_j)$ and substituted in Eq. (C.3). Finally the dispersion terms are replaced according to Eq. (4.14):

$$u(\bar{x}_i, \bar{x}_j) = \frac{s(x_i, x_j)}{m} = \frac{r(x_i, x_j) \cdot s(x_i) \cdot s(x_j)}{m} = r(x_i, x_j) \cdot \frac{s(x_i)}{\sqrt{m}} \cdot \frac{s(x_j)}{\sqrt{m}} = r(x_i, x_j) \cdot u(\bar{x}_i) \cdot u(\bar{x}_j) \tag{C.10}$$

C.3.2 Combined standard uncertainty

The combined standard uncertainty is calculated by making up all possible combinations of the two elements $c_i \cdot u(x_i)$ and $c_j \cdot u(x_j)$ including combinations with themselves and calculating the product in each case. Then, these products are totaled whereby the contribution of each product to the grand total is weighted by the respective correlation $r(x_i, x_j)$.

If the various elements $c_i \cdot u(x_i)$ are considered as being components of a vector, the calculation can be described in a systematic way as a vector equation utilizing the above matrix representations. Example for n = 3 input quantities:

$$u_C(y) = \sqrt{\begin{pmatrix} c_1 \cdot u(x_1) & c_2 \cdot u(x_2) & c_3 \cdot u(x_3) \end{pmatrix} \cdot \begin{pmatrix} 1 & r(x_1, x_2) & r(x_1, x_3) \\ r(x_2, x_1) & 1 & r(x_2, x_3) \\ r(x_3, x_1) & r(x_3, x_2) & 1 \end{pmatrix} \cdot \begin{pmatrix} c_1 \cdot u(x_1) \\ c_2 \cdot u(x_2) \\ c_3 \cdot u(x_3) \end{pmatrix}} \tag{C.11}$$

According to the rules of vector algebra the following is true for any n > 0:

$$u_C(y) = \sqrt{\sum_{i=1}^{n} c_i \cdot u(x_i) \cdot \sum_{j=1}^{n} r(x_i, x_j) \cdot c_j \cdot u(x_j)} \tag{C.12}$$

Taking account of $r(x_i, x_i) = 1$ (diagonal elements of the correlation coefficient matrix), all terms with indexes that meet the condition $i = j$ can be pooled. Then, Eq. (C.12) can be decomposed into two summation terms:

$$u_C(y) = \sqrt{\sum_{i=1}^{n} c_i \cdot u(x_i) \cdot 1 \cdot c_i \cdot u(x_i) + \sum_{i=1}^{n} c_i \cdot u(x_i) \cdot \sum_{\substack{j=1 \\ j \neq i}}^{n} r(x_i, x_j) \cdot c_j \cdot u(x_j)} \tag{C.13}$$

Taking account of the symmetry $r(x_i, x_j) = r(x_j, x_i)$, the summation in the second summation term can be restricted to the elements above the diagonal of the correlation coefficient matrix (i.e. the terms with the row index $1 \le i < n$ and column index $i + 1 \le j \le n$) if these elements are counted twice. This results in the representation according to Eq. (C.9).

D Coverage factors and degrees of freedom

D.1 Table of coverage factors k_p

Degrees of freedom ν	Confidence level $(1 - \alpha)\cdot 100\%$							
	68.2700%	90.0000%	95.0000%	95.4500%	99.0000%	99.7300%	99.9937%	99.9999%
1	1.84	6.31	12.71	13.97	63.66	235.78	10105.08	1097620.30
2	1.32	2.92	4.30	4.53	9.92	19.21	125.98	1313.06
3	1.20	2.35	3.18	3.31	5.84	9.22	32.68	156.07
4	1.14	2.13	2.78	2.87	4.60	6.62	17.47	56.68
5	1.11	2.02	2.57	2.65	4.03	5.51	12.30	31.77
6	1.09	1.94	2.45	2.52	3.71	4.90	9.85	21.98
7	1.08	1.89	2.36	2.43	3.50	4.53	8.47	17.07
8	1.07	1.86	2.31	2.37	3.36	4.28	7.60	14.23
9	1.06	1.83	2.26	2.32	3.25	4.09	7.00	12.41
10	1.05	1.81	2.23	2.28	3.17	3.96	6.57	11.15
11	1.05	1.80	2.20	2.25	3.11	3.85	6.25	10.25
12	1.04	1.78	2.18	2.23	3.05	3.76	5.99	9.56
13	1.04	1.77	2.16	2.21	3.01	3.69	5.79	9.03
14	1.04	1.76	2.14	2.20	2.98	3.64	5.62	8.61
15	1.03	1.75	2.13	2.18	2.95	3.59	5.48	8.26
16	1.03	1.75	2.12	2.17	2.92	3.54	5.37	7.97
17	1.03	1.74	2.11	2.16	2.90	3.51	5.27	7.73
18	1.03	1.73	2.10	2.15	2.88	3.48	5.18	7.52
19	1.03	1.73	2.09	2.14	2.86	3.45	5.10	7.35
20	1.03	1.72	2.09	2.13	2.85	3.42	5.04	7.19
25	1.02	1.71	2.06	2.11	2.79	3.33	4.80	6.65
30	1.02	1.70	2.04	2.09	2.75	3.27	4.65	6.32
35	1.01	1.69	2.03	2.07	2.72	3.23	4.54	6.09
40	1.01	1.68	2.02	2.06	2.70	3.20	4.47	5.94
45	1.01	1.68	2.01	2.06	2.69	3.18	4.41	5.82
50	1.01	1.68	2.01	2.05	2.68	3.16	4.37	5.73
100	1.01	1.66	1.98	2.03	2.63	3.08	4.18	5.34
1,000	1.00	1.65	1.96	2.00	2.58	3.01	4.02	5.03
10,000	1.00	1.65	1.96	2.00	2.58	3.00	4.00	5.00
100,000	1.00	1.64	1.96	2.00	2.58	3.00	4.00	5.00
∞	1.00	1.64	1.96	2.00	2.58	3.00	4.00	5.00

Table 6: Coverage factors k_p in case of normal distribution

NOTE 1: The k_p values for the degrees of freedom ν and the confidence level $(1 - \alpha)$ 100% are calculated as the (two-sided) quantiles of the t-distribution: $k_p = t_{\nu;1-\alpha/2}$ (e.g. using the EXCEL worksheet function TINV(α; ν)).

NOTE 2: If normal distribution is not applicable, other k_p factors apply (see e.g. Table 2 for triangular, rectangular and U-distribution at a confidence level of 100%; also see [GUM; G.1.3]).

D.2 Meaning of the coverage factor: Example of mean values

If the measured values $x_1, x_2, \ldots, x_k, \ldots, x_m$ with $1 \le k \le m$ were recorded for a measurand and these values are affected by random measurement errors, a more accurate estimate of the **conventional value** of the measurand is obtained according to the theory of errors by calculating the **arithmetic mean value**

$$\bar{x} = \frac{1}{m} \sum_{k=1}^{m} x_k \tag{D.1}$$

The associated **empirical standard deviation** is a measure for the dispersion of the measured values $x_1, x_2, \ldots, x_k, \ldots, x_m$ around the mean value:

$$s = \sqrt{\frac{1}{m-1} \sum_{k=1}^{m} (x_k - \bar{x})^2} \tag{D.2}$$

*NOTE 1: The quantities $\bar{x}$ and s are estimates for the parameters of a **normal distribution** which is "implicitly taken for granted" for the distribution of the measured values x_1, x_2, …, x_k, …, x_m whenever these formulae are applied.*

Without **systematic measurement errors**, i.e. if only **random measurement errors** occur, the mean value approaches the conventional value of the measurand with increasing number m of measured values and finally reaches it when m increases above all limits, i.e. when $m \to \infty$.

Since the number of measured values is always limited in practice, i.e. there is a finite number of values, the mean value also includes at least random deviations. A measure for the **mean value dispersion** to be expected in case of repeated measurements is the so-called **standard uncertainty**

$$u = \frac{s}{\sqrt{m}} = \sqrt{\frac{1}{m \cdot (m-1)} \sum_{k=1}^{m} (x_k - \bar{x})^2} \tag{D.3}$$

The uncertainty u decreases continuously as m increases and disappears when $m \to \infty$.

The expectation to discover the mean values of repeated measurements within the interval $\bar{x} - u \le \bar{x} \le \bar{x} + u$ where the **true value** of the measurand is assumed, only can be met with a certain probability. In order to quantify this probability it is necessary to specify a so-called **confidence interval**:

$$\bar{x} - t_{m-1;1-\alpha/2} \cdot u \le \bar{x} \le \bar{x} + t_{m-1;1-\alpha/2} \cdot u \tag{D.4}$$

The magnitude of the factor $t_{m-1;1-\alpha/2}$ is determined by the number m of measured values and the confidence level $1-\alpha$ which has to be specified. $t_{m-1;1-\alpha/2}$ is the (two-sided) quantile of the **t-distribution**[20] for $\nu = m-1$ **degrees of freedom** and the **confidence level** $1-\alpha$.

The confidence level 95% and m ≥ 20 measured values are common in metrology. In this case, $t_{m-1;1-\alpha/2} = 2$ applies. This means that 95 mean values of 100 (hypothetical) measurement series each consisting of 20 measured values are to be expected within the interval $\bar{x} - 2 \cdot u \le \bar{x} \le \bar{x} + 2 \cdot u$, whereby $\bar{x}$ were determined from any randomly selected measurement series out of the total of 100 measurement series.

NOTE 2: The term "hypothetical" means that these measurement series are not actually performed. In fact, an estimate of the value range is made within which the mean values of these measurement series could be expected with a certain specified probability in case the measurement series were actually performed.

In the context of measurement uncertainty studies

$$t_{m-1;1-\alpha/2} = k_p \tag{D.5}$$

is referred to as the **coverage factor** and

$$t_{m-1;1-\alpha/2} \cdot u = k_p \cdot u = U \tag{D.6}$$

as the **expanded measurement uncertainty**.

[20] Values for t as a function of ν and α can be taken from tables or determined using e.g. the MS EXCEL worksheet function TINV(α;ν); in case of EXCEL it should be noted that TINV(α;ν) directly yields the value $t_{\nu;1-\alpha/2}$.

D.3 Degrees of freedom

The coverage factor k_p is determined by the confidence level and the so-called degrees of freedom. The number of expressions in a sum minus the number of side conditions which these terms are subjected to are referred to as degrees of freedom [ISO 3534-1, 2.54].

EXAMPLE: The sum $y = x_1 + x_2 + x_3$ should lead to the same result (side condition) for all value combinations x_i. Apparently arbitrary values can be used for two of the three summands. However, the third summand must attain a certain value which is determined by the predetermined result and the other two values. Since two values x_i can be varied in any way, two degrees of freedom exist.

The "reliability" of probability data and results of statistical calculations increase with the number m of the values that contribute to the result, i.e. the better $m \to \infty$ is approximated. A limited number of values leads to side conditions for these values, i.e. a limited number of degrees of freedom.

D.3.1 Input quantities (Type A evaluation)

When determining the standard uncertainty of the input quantity i according to a type A evaluation based on m measured values that can be assumed to be normally distributed, the number of degrees of freedom is calculated as

$$\nu_i = m - 1. \tag{D.7}$$

D.3.2 Input quantities (Type B evaluation)

When determining the standard uncertainty of the input quantity i according to a type B evaluation, the following relationship can be used to estimate the number of degrees of freedom [GUM G.4.2]:

$$\nu_i = \frac{1}{2} \cdot \frac{1}{\left(\frac{\Delta u(x_i)}{u(x_i)} \right)^2} \tag{D.8}$$

The term $\Delta u(x_i)/u(x_i)$ represents the relative **uncertainty of the standard uncertainty** which affects the determined standard uncertainty $u(x_i)$, i.e. a numerical value between 0 and 1. Estimates within the range $\Delta u(x_i)/u(x_i) \leq 0{,}15$ result in $\nu_i > 20$ degrees of freedom. At a confidence level of 95.45%, $2.00 \leq k_p \leq 2.13$ or $k_p \approx 2.0$ results. At $\Delta u(x_i)/u(x_i) = 0.25$ only $\nu_i = 8$ degrees of freedom are left and $k_p \approx 2.4$ results.

In case of input quantities which can be assumed to have values lying between certain limits without exception (e.g. for physical reasons), there is no uncertainty of the uncertainty data, i.e. $\Delta u(x_i)/u(x_i) = 0$. Then, $\nu_i \to \infty$ results for the degrees of freedom. This applies e.g. to input quantities with rectangular, triangular or U-distribution according to chap. 4.4.2.2.

The situation is different for a normal distribution with values which will never lie 100% between two limits. The same applies to any other distribution if you cannot be sure of 100% of all values to lie between certain limits. In such cases a finite number of degrees of freedom exists. Unless at least 15 – 20 degrees of freedom can be assumed so that $k_p \approx 2.0$ is applicable at a confidence level of 95.45%, an analysis of the degrees of freedom is essential.

On the other hand, if the values of an input quantity i are expressly declared to be normally distributed whereas <u>no</u> limits are specified (cf. chapter 4.4.2.1), it can be shown theoretically that $m_i \to \infty$ individual measured values were needed in order to ensure with a sufficiently high confidence level (≥ 95%) that it is not a limited distribution (e.g. a triangular or a rectangular distribution which fits the data set equally well). In this case $\nu_i \to \infty$ degrees of freedom can be supposed.

D.3.3 Output quantities

For the combined standard uncertainty, the effective number of degrees of freedom can be approximated using the so-called Welch-Satterthwaite equation [GUM, G.4.1]:

$$\nu_{eff} = \frac{u_C{}^4(y)}{\sum_{i=1}^{n} \frac{(c_i \cdot u(x_i))^4}{\nu_i}} \qquad \text{or transformed} \qquad \sum_{i=1}^{n} \frac{(c_i \cdot u(x_i))^4}{\nu_i} = \frac{u_C{}^4(y)}{\nu_{eff}} \tag{D.9}$$

Input quantities i with a sufficiently large number of degrees of freedom $\nu_i \to \infty$ do not contribute to the sum. This applies e.g. to input quantities with rectangular, triangular or U-distribution according to chap. 4.4.2.2. If all n input quantities have a sufficiently large number of degrees of freedom, the effective number of degrees of freedom $\nu_{eff} \to \infty$ also results for the output quantity, and consequently $k_p \approx 2.0$ at a confidence level of 95.45%.

NOTE 1: $\nu_i \geq 15 \ldots 20$ is usually regarded as sufficiently large.

If this requirement is not met for all input quantities i, an analysis of the degrees of freedom is needed. With

$$\frac{c_i \cdot u(x_i)}{u_C(y)} = \lambda_i \tag{D.10}$$

Eq. (D.9) can be rewritten as

$$\sum_{i=1}^{n} \frac{\lambda_i{}^4}{\nu_i} = \frac{1}{\nu_{eff}} \tag{D.11}$$

which is suitable for analysis purposes since it is independent of the absolute values of uncertainty contributions.

NOTE 2: λ_i^2 represents the relative contribution of the input quantity i to the uncertainty budget (see appendix I, column "Contribution to MU budget" of the form sheet).

E Requirements of the procedures according to booklet 10 on measurement uncertainty

E.1 Allocation of capability categories

As mentioned in chap. 2.5, a sufficiently small measurement uncertainty is required so that the measurement results ensure a sufficiently reliable calculation of the parameters C_g, C_{gk} and %GRR and a corresponding assignment of the measuring process to the categories “capable”, “conditionally capable” or “not capable”.

In the case of measuring processes, the so-called “golden rule of metrology”

$$\frac{U}{T} \cdot 100\% \leq 10\%$$

represents a recommendation for the empirical upper limit of measurement uncertainty U in relation to the tolerance T of the characteristic [21]. This results in $T \geq 10 \cdot U$ or the minimum required tolerance $T_{min} = 10 \cdot U$.

The requirement of a type-1 study according to [Booklet 10]

$$C_g = \frac{0.2 \cdot T}{6 \cdot s} \geq 1.33 = \frac{4}{3}$$

results in $T \geq 40 \cdot s$ or the minimum required tolerance $T_{min} = 40 \cdot s$ for the characteristic.

The requirement of a type-2 or a type-3 study according to [Booklet 10]

$$\%GRR = \frac{6 \cdot GRR}{T} \cdot 100\% \leq 10\%$$

results in $T \geq 60 \cdot GRR$ or the minimum required tolerance $T_{min} = 60 \cdot GRR$ for the characteristic.

NOTE 1: See [Booklet 10], appendix D, for inconsistencies of the minimum requirements of a type-1 study compared to a type-2 and type-3 study.

The consequence of these three requirements is that the measurement uncertainty U must meet both conditions

$$U \leq 4 \cdot s \tag{E.1}$$

and

$$U \leq 6 \cdot GRR \tag{E.2}$$

independently of the tolerance T of the characteristic so that the measurement results allow for a reliable assessment of the measuring process.

NOTE 2: The fulfillment or non-fulfillment of these conditions does not imply any statement about “capability” or “non-capability” of the measuring process.

[21] There is no normative specification

E.2 Significance of bias according to a type-1 study, VDA volume 5 and AIAG MSA

This chapter shows (by exclusively using common definitions and limit values) that a meaningful significance test is linked to certain requirements on measurement uncertainty.

Definitions

Capability index: $C_g = \frac{0.2 \cdot T}{6 \cdot s}$

Minimum capability index: $C_{gk} = \frac{0.1 \cdot T - |\bar{x} - x_0|}{3 \cdot s}$

Step 1

The definition for C_g utilized in the definition for C_{gk}

$$C_{gk} = \frac{0.1 \cdot T}{3 \cdot s} - \frac{|\bar{x} - x_0|}{3 \cdot s} = \frac{0.2 \cdot T}{6 \cdot s} - \frac{|\bar{x} - x_0|}{3 \cdot s} = C_g - \frac{|\bar{x} - x_0|}{3 \cdot s}$$

and solved for $C_g - C_{gk}$ yields

$$C_g - C_{gk} = \frac{1}{3} \cdot \frac{|\bar{x} - x_0|}{s}.$$

The systematic measurement error $|\bar{x} - x_0|$ of a sample of size m related to the standard deviation s is insignificant at a confidence level of 1 - α if

$$\frac{|\bar{x} - x_0|}{s} \le \frac{t_{m-1;1-\alpha/2}}{\sqrt{m}}$$

(see [Booklet 10], appendix C), i.e. if

$$\boxed{C_g - C_{gk} \le \frac{1}{3} \cdot \frac{t_{m-1;1-\alpha/2}}{\sqrt{m}}}. \tag{E.3}$$

$t_{m-1;1-\alpha/2}$ denotes the quantile of the t-distribution for m – 1 degrees of freedom and a confidence level 1 – α and a confidence interval limited on both sides.

Step 2

The definition for C_g solved for 3s

$$3 \cdot s = \frac{0{,}1 \cdot T}{C_g}$$

substituted in the definition of C_{gk}

$$C_{gk} = \frac{0{,}1 \cdot T - |\bar{x} - x_0|}{\frac{0{,}1 \cdot T}{C_g}} = C_g - 10 \cdot C_g \cdot \frac{|\bar{x} - x_0|}{T},$$

solved for $C_g - C_{gk}$ and taking account of Eq. (E.3) from step 1 yields

$$\boxed{C_g - C_{gk} = 10 \cdot C_g \cdot \frac{|\bar{x} - x_0|}{T} \le \frac{1}{3} \cdot \frac{t_{m-1;1-\alpha/2}}{\sqrt{m}}} \tag{E.4}$$

The inequality (E.4) solved for $|\bar{x} - x_0|/T$ finally results in

$$\boxed{\frac{|\bar{x} - x_0|}{T} \le \frac{1}{30 \cdot C_g} \cdot \frac{t_{m-1;1-\alpha/2}}{\sqrt{m}}} \tag{E.5}$$

Result

In terms of figures, a confidence level of 95%, usual sample size (m = 25 ... 50) and capability in the range $C_g \geq 1.33$ result in the requirement

$$\frac{|\bar{x} - x_0|}{T} \cdot 100\% < 1\%.$$

This means that the systematic measurement error of a measuring system must not be greater than 1% of the tolerance of the characteristic in order to be considered insignificant.

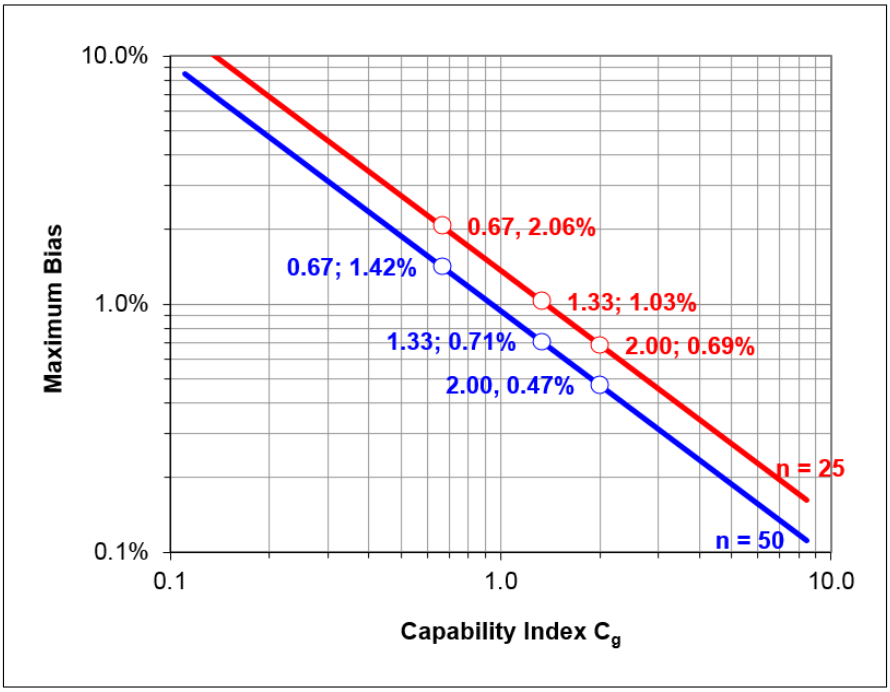

Figure 9: Limit values for insignificant systematic measurement error with a type-1 study
Maximum values related to the characteristic tolerance at a confidence level of 95% shown for the sample sizes m = 50 and m = 25 dependent on the capability index C_g of the measuring process.

Significance for practical application

In practice, a significance test of this type is only relevant if the measurement uncertainty U meets the condition

$$\frac{U}{T} \cdot 100\% < 1\% \,.$$

This is often not achieved. Instead, the so-called "golden rule of metrology" is considered to be the rule of thumb for "suitable" measuring systems,

$$\frac{U}{T} \cdot 100\% < 10\% \,,$$

i.e. a requirement reduced by a factor 10. It should be also noted that the upper limit of 10% represents a limit which has proven itself empirically, however, which is not clearly defined in guidelines and standards. Depending on the measuring system, it may happen that limit values up to approximately 20% are acceptable.
It should be noted as well that the measurement uncertainty U never can be less than the resolution of the measuring system.

Conclusion

In the case

$$\frac{|\bar{x} - x_0|}{T} < \frac{U}{T} \qquad \text{or simply} \qquad |\bar{x} - x_0| < U$$

the evaluation of the systematic measurement error of a measuring system by means of a significance test is not useful, since it is within the range of measurement uncertainty (value range for the true value of the measurement result). Then, it is not possible to decide whether the test result represents a purely computational result or an actual technical deviation.

F Consideration of systematic measurement errors (correction)

***NOTE**: Numerous guidelines provide approaches more or less different from [GUM]. The approach described below is directly based on [GUM, H.3].*

F.1 Uncertainty of the corrected measurement result

Provided that a linear correction of an observed measurement result y' (indication) is sufficient, the following model equation is applied to describe the relationship with the corrected measurement result y_0 (conventional value, "correct" value):

$$y_0 = y' + K(y') \tag{F.1}$$

with the correction

$$K(y') = y_0 - y' = \alpha_K + \beta_K \cdot y' \tag{F.2}$$

NOTE 1: α_K and β_K are parameters of the correction curve, i.e. they usually represent the intercept and slope of a regression line. This line is determined e.g. as part of a calibration using several standards with different reference values (see appendix F.2 and [GUM, H.3]).

NOTE 2: For non-linear corrections, e.g. by means of higher order polynomials, specialist literature should be referred to.

When determining the uncertainty $u(y_0)$ of the corrected measurement result $y_0 = y_0(\alpha_K, \beta_K, y')$, only the **uncertainty** $u(K(y'))$ of the correction $K(y')$ must be considered but not the correction itself. With the sensitivity coefficients

$$\frac{\partial K}{\partial \alpha_K} = 1, \qquad \frac{\partial K}{\partial \beta_K} = y' \qquad \text{and} \qquad \frac{\partial K}{\partial y'} = \beta_K \tag{F.3}$$

the uncertainty of the correction is calculated according to

$$u(K(y')) = \sqrt{u^2(\alpha_K) + y'^2 \cdot u^2(\beta_K) + \beta_K^2 \cdot u^2(y') + 2 \cdot y' \cdot u(\alpha_K, \beta_K)}\,. \tag{F.4}$$

NOTE 3: It is essential to note that the regression coefficients α_K and β_K normally are determined from the same measurement data set and therefore they are correlated (see appendix C). Usually, this contribution is not negligible and must be taken into account in the uncertainty analysis [EUROLAB, A.2.1]. The term $2 \cdot y' \cdot u(\alpha_K, \beta_K)$ represents this correlation.

Accordingly, the following applies to the uncertainty of the corrected measurement result:

$$u(y_0) = \sqrt{u^2(y') + u^2(K(y'))} \tag{F.5}$$

__Practical special cases__

- $\beta_K = 0$, i.e. a constant additive correction according to $y_0 = y' + \alpha_K$ which is independent of the measured value:

$$u(y_0) = \sqrt{u^2(\alpha_K) + u^2(y')} \tag{F.6}$$

- $\alpha_K = 0$, i.e. a correction by a constant factor relative the measured value according to $y_0 = (1 + \beta_K) \cdot y'$:

$$u(y_0) = \sqrt{y'^2 \cdot u^2(\beta_K) + (1 + \beta_K^2) \cdot u^2(y')} \tag{F.7}$$

F.2 Correction and correction uncertainty in case of linear regression

The parameters α_K and β_K for the correction $K(y')$ and its standard uncertainty $u(K(y'))$ are usually determined by means of several standards with different reference values $x_{0,j}$ and the associated values x'_j indicated by the measuring system. The evaluation is performed with the aid of Eq. (F.2) where x_0 takes the place of y_0 and x' takes the place of y':

$$K(x') = x_0 - x' = \alpha_K + \beta_K \cdot x' \tag{F.8}$$

The following Eqs. (F.9) to (F.14) are standard relationships which can be taken e.g. from textbooks covering linear regression (see also [GUM, H.3] and [EUROLAB, A.2]). The nomenclature has been adapted to the present case. The applicability assumes insignificant effects of the **reading uncertainty** and the **calibration uncertainty** of the standards on the uncertainty of the calculated parameters and a sufficiently constant **residual dispersion** s_R around the regression line. Otherwise, different relationships apply. The technical literature should be referred to for this point.

Observed correction at standard j:

$$K_j = x_{0,j} - x'_j \tag{F.9}$$

Mean values of the observed correction values and the measured values for n_0 standards:

$$\overline{K} = \frac{1}{n_0}\sum_{j=1}^{n_0} K_j \qquad \overline{x'} = \frac{1}{n_0}\sum_{j=1}^{n_0} x'_j \tag{F.10}$$

Variance of the observed measured values and covariance of the observed measured values and the correction values, each multiplied by the factor $(n_0 - 1)$:

$$Q_{x'} = \sum_{j=1}^{n_0}\left(x'_j - \overline{x'}\right)^2 \qquad Q_K = \sum_{j=1}^{n_0}\left(x'_j - \overline{x'}\right)\cdot\left(K_j - \overline{K}\right) \tag{F.11}$$

Slope and intercept of the regression line:

$$\beta_K = \frac{Q_K}{Q_{x'}} \qquad \alpha_K = \overline{K} - \beta_K \cdot \overline{x'} \tag{F.12}$$

Residual dispersion of the observed correction values around the regression line:

$$s_R^{\ 2} = \frac{\sum_{j=1}^{n_0}\left\{K_j - \left(\alpha_K + \beta_K \cdot x'_j\right)\right\}^2}{n_0 - 2} \tag{F.13}$$

Variance and covariance of intercept and slope of the regression line:

$$u^2(\alpha_K) = s_R^{\ 2}\cdot\left(\frac{1}{n_0} + \frac{\overline{x'}^2}{Q_{x'}}\right) \qquad u^2(\beta_K) = s_R^{\ 2}\cdot\frac{1}{Q_{x'}} \qquad u(\alpha_K,\beta_K) = -s_R^{\ 2}\cdot\frac{\overline{x'}}{Q_{x'}} \tag{F.14}$$

In order to determine the correction $K(y')$ of any measurement result y' observed within the range $MIN(x_{0,j}) \leq y' \leq MAX(x_{0,j})$, the parameter values determined according to the Eqs. (F.12) for the intercept α_K and the slope β_K of the regression line are substituted in Eq. (F.2). To determine the uncertainty of the correction $u(K(y'))$, the parameter values determined according to the Eqs. (F.14) for the variances $u^2(\alpha_K)$ and $u^2(\beta_K)$ and the covariance $u(\alpha_K,\beta_K)$ of the intercept and slope are substituted in Eq. (F.4).

F.3 Uncertainty of the uncorrected measurement result

> ***NOTE**: According to [GUM, 6.3.1, note] measurement results not being corrected although the required corrections are known have to be avoided in general. Sometimes, however, this case cannot be avoided (see [GUM, F.2.4.5]). Even so, it must be restricted to special circumstances, substantially reasoned and documented.*

If the measurement result is not corrected y' despite the correction $K(y')$ is known, both the uncertainty $u(K(y'))$ of the correction as well as the correction $K(y')$ itself must be taken into account as uncertainty components in the uncertainty $u^*(y')$ of the **uncorrected** measurement result (see [EUROLAB], chap. 4) [22] :

$$u^*(y') = \sqrt{u^2(y') + u^2(K(y')) + K^2(y')} = \sqrt{u^2(y_0) + K^2(y')} \tag{F.15}$$

[22] See also I.H.Lira, W.Wöger, Meas. Sci. Technol. **9** (1998), 1010-1011 as well as *"Erklärung der PTB zur Behandlung systematischer Abweichungen bei der Berechnung der Messunsicherheit"* (2010-05-12)

G Comparability of measurement results

For the purpose of evaluating the comparability of measurement results of different laboratories and measuring instruments, the *European Cooperation for Accreditation (EA)* suggested using the parameter E_n [ISO 17043; ISO 13528]:

$$E_n = \frac{y_{LAB} - y_{REF}}{\sqrt{U_{LAB}^2 + U_{REF}^2}} \qquad \text{(G.1)}$$

with

y_{LAB}	Measurement result of the laboratory considered,
U_{LAB}	Associated expanded measurement uncertainty of the laboratory considered,
y_{REF}	Reference value of a higher-level laboratory (e.g. PTB, NIST, NPL),
U_{REF}	Associated expanded measurement uncertainty of the higher-level laboratory.

The comparability of the measurement results will be classified as acceptable if the criterion $E_n \le 1$ is met. In the case $E_n > 1$ corrective and possibly monitoring measures are required.

NOTE: The applicability of this parameter is not restricted to different laboratories. It can be applied equally to several measuring systems of the same laboratory, for example. Application to several measurement results of the same measuring system is also possible.

If a reference value y_{REF} of a higher-level laboratory with significantly smaller measurement uncertainty U_{REF} is unavailable, the mean value of the measurement results of all laboratories concerned can be used as a reference value y_{REF}:

$$y_{REF} = \overline{y_{LAB}} = \frac{1}{N_{LAB}} \cdot \sum_{N=1}^{N_{LAB}} y_{LAB_N} \qquad \text{(G.2)}$$

with

y_{LAB_N}	Measurement result of laboratory no. N,
N_{LAB}	Total number of laboratories concerned.

Accordingly, U_{REF} is calculated from the average of the variances of the standard uncertainties of all laboratories concerned:

$$U_{REF} = \overline{U_{LAB}} = k_p \cdot \sqrt{\frac{1}{N_{LAB}} \sum_{N=1}^{N_{LAB}} \left(\frac{U_{LAB_N}}{k_{p\,LAB_N}} \right)^2} = k_p \cdot \sqrt{\overline{u_{LAB}^2}} \qquad \text{(G.3)}$$

with

U_{LAB_N}	Expanded measurement uncertainty for the measurement result of laboratory no. N,
$k_{p\,LAB_N}$	Coverage factor of the expanded measurement uncertainty of laboratory no. N,
k_p	Coverage factor of the expanded reference uncertainty.

Moreover, Eq. (G.1) enables a criterion for the distinctness of measurement results from the same measuring system to be defined in an alternative way to chap. 2.3. According to Eq. (G.1) the measurement results y_1 and y_2 are different if

$$\frac{y_2 - y_1}{\sqrt{U_2^2 + U_1^2}} > 1 \qquad \text{(G.4)}$$

is fulfilled. Since the measuring results were obtained with the same measuring system, $U_2 = U_1 = U$ can be assumed so that

$$y_2 - y_1 > \sqrt{2} \cdot U \qquad \text{(G.5)}$$

results as a criterion for distinguishable values y_1 and y_2.

H Monte Carlo simulation

It is not always possible to determine the measurement uncertainty of a measurand with reasonable effort in an analytical way based on the Gaussian error propagation law, i.e. through the manual analysis of a mathematical model equation. The effort for calculating partial derivatives or output quantity values can increase dramatically, in particular in the case of complex (e.g. nonlinear) mathematical relationships.

In such cases, the Monte Carlo simulation method provides an alternative. Based on stochastics (probability theory) and by means of random numbers, this method simulates the impact of the variation of the input quantities (variables) of a mathematical formula (model equation) on the output quantity (result).

Accordingly, just as with manual analysis, it is a basic prerequisite for the Monte Carlo method that the functional relationship (model equation) between the input quantities and the output quantity (cf. chapter 4.3) is available. Furthermore, knowledge about the target value or the expected value and the distribution model of associated input values around the target value or the expected value is required for each input quantity. The input values may be estimated or measured values which represent the practical application as closely to reality as possible.

Unlike manual evaluations, highly complex relationships which cannot be described by means of a single analytical equation and unusual distributions of input values are possible. Examples are:

- Absolute value,
- Hysteresis,
- limited range (“clipping”, e.g. in the case of limited frequency bands),
- Idle time,
- Backlash (e.g. differences of coordinate measuring machines when approaching a measuring point from the left or the right)
- Constraint (e.g. overcoming frictional resistance)
- Interpolation using predetermined points.

The simulation is performed with estimated or measured values being used for each input quantity and each individual value being varied randomly according to the established distribution models. A sufficiently large number of simulation runs provides assertions about the dispersion range and distribution of the output values.

Please, refer to [GUM-S1] for details.

I Form for tabular uncertainty budgets

Information about input quantities							Standard uncertainties of input quantities				Contributions to the measurement uncertainty of the measurand					Determining k_p for the measurand		
Seq. No.	Description	Variable (symbol)	Measuring unit	Value of the variable	Value of the uncertainty data	Comments (z.B. references, explanatory notes, links to documents)	Evaluation type	Type A: Number of measured values; Type B: kp (≥1), confidence level (%), distribution	Numerical factor for calculating the standard uncertainty	Standard uncertainty	Sensitivity coefficient	Contribution to uncertainty	Contribution to uncertainty (squared)	Percentage contribution to MU budget $\frac{(c_i \cdot u(x_i))^2}{\sum_{i=1}^{n}(c_i \cdot u(x_i))^2}$	Rank (according to Pareto)	Estimated uncertainty of the uncertainty data	Degrees of freedom	Contribution to the denominator of the Welch-Satterthwaite formula
i				x_i	Δx_i		A B	m_i k_p, %, name	1 or $\frac{\sqrt{m_i}}{k_p}$	$u(x_i) = \Delta x_i / k_p$	c_i	$c_i * u(x_i)$	$(c_i * u(x_i))^2$	[%]		$\Delta u(x_i) / u(x_i)$	ν_i	$\frac{(c_i \cdot u(x_i))^4}{\nu_i}$
1																		
2																		
3																		
4																		
5																		
Model equation:							Total result:				$u_c^2 =$					$\sum_i =$		
											$u_c =$					$\nu_{eff} =$		
											$k_p =$					$1 - \alpha =$		
											$U =$					$k_p =$		

Table 7: Form sheet “Tabular uncertainty budget” form (proposal)

The form contains four groups of columns with the following contents:

- **Information about input quantities:** Systematic documentation of all available information about the input quantities;
- **Standard uncertainties of input quantities:** Calculation from the available information;
- **Contributions to the measurement uncertainty of the measurand:** Calculation from the standard uncertainties;
- **Determining k_p for the measurand.**

Every **row of the table** refers to a specific input quantity.

One or more auxiliary lines containing intermediate and auxiliary calculations for this input quantity can be inserted above each table line. Auxiliary lines do not include a "seq. no." and only contain data in the column group "Information about input quantities" . All other columns remain empty [23] .

Information about input quantities

Column heading	**Column content**
Seq. no.	Integer
Description	Unique identifier (name) of the input quantity, e.g. "bracket length" *(i.e. not merely unspecific "length")*
Variable (symbol)	Symbol for the input quantity, e.g. L_B *(i.e. L for "length" and "B" for bracket)*
Measuring unit	Measuring unit of the numeric value of the input quantity and the associated uncertainty data *(e.g. m for meters)*
Value of the variable	Numerical value of the input quantity *(e.g. 7.5)*
Value of the uncertainty data	Numerical value of the uncertainty data *(e.g. 0.02)*
Comments (e.g. …)	Free text, e.g. sources, notes, calculation formulas, references, links to documents

Standard uncertainties of input quantities

Column heading	**Column content**
Evaluation type	A or B in accordance with the evaluation type used for the standard uncertainty of the corresponding input quantity
Type A: Number of measured values Type B: k_p (≥ 1), confidence level (%), distribution	• Type A: Unspecified or integer ≥ 1 • Type B: Value ≥ 1 ***or*** confidence level *(Percentage between 0% and 100%)* ***or*** designation of the distribution model *(e.g. triangular distribution)*
Numerical factor for calculating the standard uncertainty	Numerical value by which the uncertainty data associated with the input quantity are divided to determine the standard uncertainty: • Type A: 1 or $\sqrt{m}$ • Type B: k_p
Standard uncertainty	Determined standard uncertainty of the input quantity

[23] For exceptions, see tables in examples J.5.1 and J.5.2

Contributions to the measurement uncertainty of the output quantity

Column heading	Column content
Sensitivity coefficient	Numerical value of the sensitivity coefficient of the corresponding input quantity
Contribution to uncertainty	Standard uncertainty multiplied by the sensitivity coefficients
Contribution to uncertainty (squared)	Numerical value of the column “contribution to uncertainty” multiplied by itself
Percentage contribution to MU budget	Numerical value of the column “contribution to uncertainty (squared)” as a percentage of the grand total of this column
Rank (according to Pareto)	Numerical values sorted by decreasing quantity, i.e. rank 1 has the highest significance, rank 2 has the second highest, etc.

Determining k_p for the output quantity (optional)

Column heading	Column content
Estimated uncertainty of the uncertainty data	Numerical value as a percentage (see appendix D.3.2)
Degrees of freedom	Integer (see appendix D.3 for details)
Contribution to the denominator of the Welch-Satterthwaite formula	Numerical value (for further details, see appendix D.3.3)

J Examples

With the exception of the "folding ruler" example, examples from real life are used in all cases. Simplifications are only made in some cases where not all possible input quantities are considered (e.g. uncertainties of material parameters such as the thermal coefficient of expansion).

- J.1 Marking using a folding ruler (coll. yardstick)
  Simple illustration of the basic procedure of a measurement uncertainty study using the example of length and surface markings; application of the additive and multiplicative model, normal and triangular distribution and consideration of correlations.
- J.2 Evaluating the suitability of a dial gauge
  Determining the uncertainty of the measurement results of a dial gauge that is calibrated for the special application of testing a specific product characteristic for compliance with a (fixed) specification; application of the additive model, avoidance of corrections.
- J.3 Measuring a bolt diameter
  Determining the uncertainty of measurement results for bolt diameters; application of corrections and degrees of freedom (input quantities of type A and B, Welch-Satterthwaite formula).
- J.4 Torque measurement using an engine test
  Determining the uncertainty of torque measurement results based exclusively on the manufacturer's specifications, calibration certificates and experience (type B evaluation, no measurements).
- J.5 Optical measurement using a measuring microscope
  Determining and assessing the uncertainty of visually determined measurement results in accordance with ISO 22514-7.
- J.6 In-process tactile diameter measurement
  Determining the uncertainty of the measurement results of a measuring process based on stability charts.
- J.7 Injection quantity indicator (EMI)
  More sophisticated practical example: Uncertainty of the calibration of a measuring system based on a closed-form mathematical model; establishing the model equation, non-linear correction, uncertainty of the correction, using sensitivity coefficients.
- J.8 Pressure sensor
  More sophisticated practical example: Determining correction and measurement uncertainty using a "mixed" model (additive overall model with closed-form mathematical submodel) for direct use in practical applications; impact of corrections that are not made; impact of use outside the calibrated temperature range.

The aim is to illustrate the determination (calculation) of measurement uncertainties by means of real-life data (numerical values) in a clear, comprehensible and reproducible way. Therefore, all information is waived that is not essential for determining the measurement uncertainty. However, it is expressly pointed out that the full documentation of a measurement uncertainty study must include at least the following information:

- unique identification of the measuring system (e.g. location, department, measuring system designation, inventory number, serial number);
- date and time of the beginning and end of each measurement with indication of relevant environmental conditions (such as ambient temperature, humidity, air pressure and light intensity);
- unique identification of the operators (operating, checking and analyzing) and the persons in charge by means of ID codes or names (*note that uncoded names are not allowed in all countries)*;
- any particular incidents during the measurement where applicable;
- clear references to related documents (e.g. ID number, designation, version, date).

J.1 Marking using a folding ruler (coll. yardstick)

Lengths and area sections should be marked. Commercially available folding rulers (coll. “yardstick”, see Figure 10) with the following characteristics are used for this purpose:

- Total length of the ruler L_T = 2 m = 2000 mm,
- Length of a ruler element L_E = 20 cm = 200 mm,
- Scale spacing L_S = 1 mm.

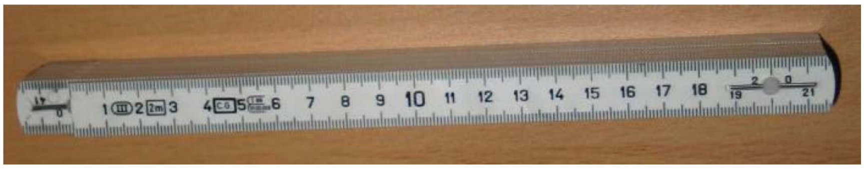

Figure 10: Commercially available folding ruler (accuracy class III, total length 2 m)

According to its labeling the ruler is of accuracy class III. Thus, the maximum permissible measurement error (“error limit”) in mm is calculated according to the formula [24]:

$$\delta L_L \le \delta L_{MAX} = 0.6 + 0.4 \cdot L^* .$$

For L^* the numerical value has to be substituted which results from rounding up the length L_0 to be measured to the next full meter (e.g. L^* = 1 for the length L_0 = 0.30 m to be measured, L^* = 2 for the length L_0 = 1.75 m to be measured).

*NOTE: In order to present the basic procedure as simple as possible, only those uncertainties are considered that are caused by the folding ruler itself. Other uncertainties such as arising from placing the ruler in position against certain datum points, marking of the desired position, squareness and position of the 4th corner point when marking surfaces are **not** considered in this example. In order to take account of these additional uncertainties, appropriate input quantities need to be identified and included.*

J.1.1 Marking two points at a distance up to the length of one ruler element

Description of the measurement

A second point should be marked at a distance of L_0 = 15 cm from a predetermined point. The marking is done by simply applying and measuring using one ruler element.

Input quantities

- Nominal value of the length to be measured $L_0 = 150$ mm

Model

$L = L_0 + \delta L_L$

with

L	Actual value of the measured length,
L_0	Nominal value of the measured length (no uncertainty),
δL_L	Deviation due to the limited accuracy of the total ruler length.

[24] According to “*Directive 2004/22/EC of the European Parliament and Council of March 31, 2004, on measuring instruments*”, “*Appendix MI-008 Material Measures*”, *table 1*

Standard uncertainties of the input quantities

- The maximum permissible measurement deviation may lead to variations within the limits

  $a_+ = +\delta L_{MAX}$ and $a_- = -\delta L_{MAX}$

  i.e. cause the maximum deviation

  $$a = \frac{a_+ - a_-}{2} = \frac{\delta L_{MAX} - (-\delta L_{MAX})}{2} = \frac{2 \cdot \delta L_{MAX}}{2} = \delta L_{MAX} = (0.6 + 0.4 \cdot L^*)\,\text{mm} = (0.6 + 0.4 \cdot 1)\,\text{mm} = 1.0\,\text{mm}$$

  As explained above, L* = 1 is used since the length to be measured is L_0 = 0.15 m. Assuming a normal distribution the standard uncertainty is

  $$u_L = \frac{a}{2} = \frac{1.0}{2}\,\text{mm} = 0.5\,\text{mm}$$

- Further input quantities are considered to be insignificant (see introduction, chap. J.1, note).

Standard uncertainty of the output quantity

Since only one input quantity is taken into account, it is likewise the output quantity :

$$u_C = u_L = 0.5\,\text{mm}$$

Expanded measurement uncertainty

The expanded measurement uncertainty is calculated using k_p = 2:

$$U = k_p \cdot u_C = 2 \cdot 0.5\,\text{mm} = 1.0\,\text{mm} = 0.1\,\text{cm}$$

Complete measurement result

$$L \pm U = (15.0 \pm 0.1)\,\text{cm}.$$

Accordingly, a marking at a nominal distance L_0 = 15 cm from a specified point is actually located in the range between L = 14.9 cm and L = 15.1 cm with a confidence level of 95.45% (corresponding to k_p = 2).

J.1.2 Marking two points at a distance of several lengths of a ruler element

Description of the measurement

A second point should be marked at a distance of L_0 = 150 cm from a predetermined point. The marking is done by simply applying and measuring using several ruler elements.

Input quantities

- Nominal value of the length to be measured $L_0 = 1500\,\text{mm}$
- Locking mechanism between the ruler elements (see Figure 12):
  - Distance between link axis and center of the bevelled edge area $s = 12\,\text{mm}$
  - Width of the bevelled edge area $\Delta s = 1\,\text{mm}$
- Length of a single ruler element $L_E = 200\,\text{mm}$

Model

$$L = L_0 + \delta L_L + n_E \cdot \delta L_\varphi$$

with

L	Actual value of the measured length,
L_0	Nominal value of the measured length (no uncertainty),
δL_L	Deviation due to the limited accuracy of the total ruler length,
n_E	Required number of ruler elements (decimal digit),
δL_φ	Deviation due to limited accuracy of the alignment of two ruler elements in an exactly straight line.

Standard uncertainties of the input quantities

- The maximum permissible measurement deviation may lead to variations within the limits

  $a_+ = +\delta L_{MAX}$ and $a_- = -\delta L_{MAX}$

  i.e. cause the maximum deviation

  $$a = \frac{a_+ - a_-}{2} = \frac{\delta L_{MAX} - (-\delta L_{MAX})}{2} = \frac{2 \cdot \delta L_{MAX}}{2} = \delta L_{MAX} = (0.6 + 0.4 \cdot L^*)\,\text{mm} = (0.6 + 0.4 \cdot 2)\,\text{mm} = 1.4\,\text{mm}$$

  As explained above, $L^* = 2$ is used since the length to be measured is $L_0 = 1.5$ m. Assuming a normal distribution the standard uncertainty is

  $$u_L = \frac{a}{2} = \frac{1.4}{2}\,\text{mm} = 0.7\,\text{mm}$$

- The measurement requires the application of several ruler elements. Therefore, an angle φ between the individual elements of the ruler must be considered which leads to a deviation from the exact straightness of the ruler and thereby to a shortening of the actually measured length L compared to its nominal value L_0 (see Figure 11):

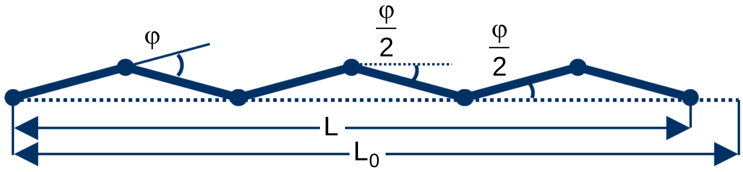

Figure 11: Deviations of the applied folding ruler from precise straightness

The angle between two elements is caused by the backlash of the locking which is mainly due to the bevelled step at the edge of the locking mechanism (see Figure 12).

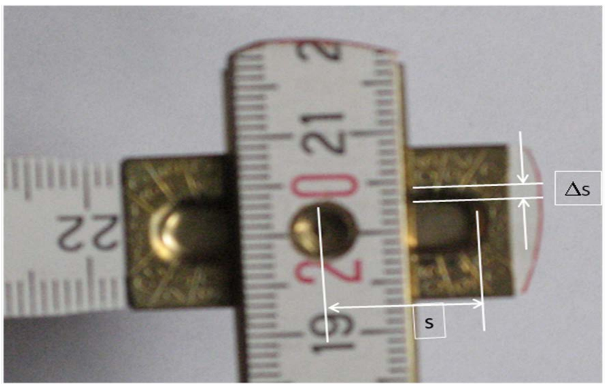

Figure 12: Folding ruler, link and locking mechnism between ruler elements

The bevel width is Δs = 1 mm. Δs related to the distance s = 12 mm between the link axis and the center of the beveled edge area yields the following relationship for the maximum angle φ:

$$\tan\varphi = \frac{\Delta s}{s} = \frac{1\,\text{mm}}{12\,\text{mm}} \approx 0.083333 \qquad \text{or} \qquad \varphi = \arctan\left(\frac{\Delta s}{s}\right) \approx \arctan\left(\frac{1\text{mm}}{12\,\text{mm}}\right) \approx 0.083141$$

or $\varphi \approx 4{,}764°$ when converted from radian measure into angular measure.

NOTE: Conversion by multiplying by 360° / (2π) ≈57.296°.

In relation to the ideally straight line between the start and end point of the length to be measured, the deviations $\delta\varphi$ vary in the range

$$-\frac{\varphi}{2} \le \delta\varphi \le +\frac{\varphi}{2}$$

(see Figure 11). These deviations can cause a shortening of the actually gauged length up to

$$\delta L_\varphi = L_E - L_E \cdot \cos\frac{\varphi}{2} = \left(1 - \cos\frac{\varphi}{2}\right) \cdot L_E$$

for *each ruler element* which, however, actually contributes its nominal value L_E to the measurement result (see Figure 13).

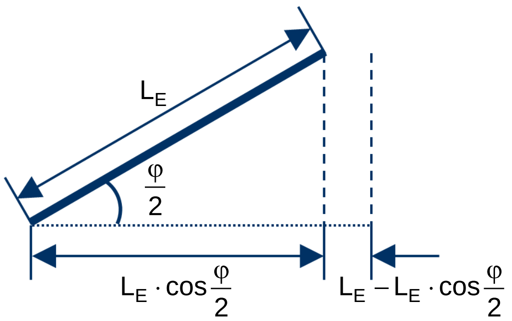

Figure 13: Deviation of the length measurement due to angle deviation

The approximation $\cos\frac{\varphi}{2} \approx 1 - \frac{1}{2} \cdot \left(\frac{\varphi}{2}\right)^2$ applies to small angles, so that

$$\delta L_\varphi \approx \frac{\varphi^2}{8} \cdot L_E \; .$$

Per ruler element, this uncertainty of the alignment can lead to a deviation of the actually marked length within the limits

$a_+ = +\delta L_\varphi$ and $a_- = -\delta L_\varphi$

i.e. the maximum deviation is

$$a = \frac{a_+ - a_-}{2} = \frac{\delta L_\varphi - \left(-\delta L_\varphi\right)}{2} = \frac{2 \cdot \delta L_\varphi}{2} = \delta L_\varphi \approx \frac{\varphi^2}{8} \cdot L_E = \frac{0.083141^2}{8} \cdot 200\text{ mm} \approx 0.173\text{ mm} \, .$$

The total length $L_0 = 150\text{ cm}$ to be measured requires

$$n_E = L_0 / L_E = 150\text{ cm} / 20\text{ cm} = 7.5$$

ruler elements, i.e. 7 complete elements and half of the 8th element. So, assuming a triangular distribution as an approximation for a limited, i.e. truncated normal distribution (exceeding the limit value is impossible for mechanical reasons), the following standard uncertainty caused by angular deviations results for the total length to be gauged:

$$u_\varphi = n_E \cdot \frac{a}{\sqrt{6}} = 7.5 \cdot \frac{0.173\,\text{mm}}{2.449} \approx 0.529\text{ mm} \, .$$

- Further input quantities are considered to be insignificant (see introduction, chap. J.1, note).

Standard uncertainty of the output quantity

$$u_C = \sqrt{u_L^2 + u_\varphi^2} \approx \sqrt{0.7^2\ \text{mm}^2 + 0.529^2\ \text{mm}^2} \approx \sqrt{0.49 + 0.279973}\ \text{mm} = \sqrt{0.769973}\ \text{mm} \approx 0.877\ \text{mm}$$

Expanded measurement uncertainty

The expanded measurement uncertainty is calculated using $k_p = 2$:

$$U = k_p \cdot u_C = 2 \cdot 0.877\ \text{mm} = 1.754\ \text{mm} \approx 0.18\ \text{cm}$$

Complete measurement result

$$L \pm U = (150.00 \pm 0.18)\ \text{cm}$$

Accordingly, a marking at a nominal distance L_0 = 150 cm from a specified point is actually located in the range between L = 149.82 cm and L = 150.18 cm with a confidence level of 95.45% (corresponding to k_p = 2).

J.1.3 Marking an area using two folding rulers

Description of the measurement

A rectangular area with the edge lengths L_{0x} = 15 cm and L_{0y} = 150 cm shall be marked. Marking is done by applying and measuring using two **different** rulers. One ruler is used for the x-direction, the other one is used for the y-direction.

Input quantities

- Short side (edge length $L_{0x} = 15\ \text{cm}$): cf. chapter J.1.1
- Long side (edge length $L_{0y} = 150\ \text{cm}$): cf. chapter J.1.2

Model

$$A = L_x \cdot L_y = (L_{0x} + \delta L_x) \cdot (L_{0y} + \delta L_y + n_E \cdot \delta L_\varphi) \tag{J.1}$$

with

A	Actual value of the marked area,
L_x, L_y	Actual values of the measured lengths in the x-direction or the y-direction,
L_{0x}, L_{0y}	Nominal values of the measured lengths in the x-direction or the y-direction (conventional values, no uncertainty),
$\delta L_x, \delta L_y$	Deviations in the x-direction or the y-direction due to the limited accuracy of the total ruler length,
n_E	Required number of ruler elements (decimal number),
δL_φ	Deviation due to the limited accuracy of alignment of two ruler elements in an exactly straight line.

Standard uncertainties of the input quantities

- The edge length to be measured in the x-direction is L_{0x} = 0.15 m. Therefore, the standard uncertainty determined in chap. J.1.1 applies to the x-direction:

  $u_x = u_C = 0.5\ \text{mm}$
- The edge length to be measured in the y direction is L_{0y} = 1.50 m. Therefore, the standard uncertainty determined in chap. J.1.2 applies to the y-direction:

  $u_y = u_C = 0.877\ \text{mm}$
- Further input quantities are considered to be insignificant (see introduction chap. J.1, note).

Standard uncertainty of the output quantity

In case of multiplicative models such as Eq. (J.1) the combined standard uncertainty of the output quantity can be determined from the following relationship (cf. chapter 4.5):

$$\frac{u_C}{A} = \sqrt{\left(\frac{u_x}{L_x}\right)^2 + \left(\frac{u_y}{L_y}\right)^2} \approx \sqrt{\left(\frac{0.5\ \text{mm}}{150\ \text{mm}}\right)^2 + \left(\frac{0.877\ \text{mm}}{1500\ \text{mm}}\right)^2} \approx 0.003384$$

Thus, the standard uncertainty u_C of the area

$$A = L_x \cdot L_y = 150\ \text{mm} \cdot 1500\ \text{mm} = 225000\ \text{mm}^2 = 2250\ \text{cm}^2$$

is

$$u_C = 0.003384 \cdot 225000\ \text{mm}^2 = 761.4\ \text{mm}^2 \approx 7.6\ \text{cm}^2$$

Expanded measurement uncertainty

The expanded measurement uncertainty is calculated using k_p = 2:

$$U = k_p \cdot u_C = 2 \cdot 761.4\ \text{mm}^2 = 1522.8\ \text{mm}^2 \approx 15.2\ \text{cm}^2$$

Complete measurement result

$$A \pm U = (2250.0 \pm 15.2)\ \text{cm}^2$$

Accordingly, in case of marking a rectangular area of nominal size A = 2250 cm^2, the actual size of the marked area ranges between A = 2234.8 cm^2 and A = 2265.2 cm^2 with a confidence interval of 95.45% (according to k_p = 2). These are approximately 0.68% uncertainty in relation to the nominal size.

J.1.4 Marking an area using a single folding ruler

Description of the measurement

The task is exactly the same as in chap. J.1.3: A rectangular area section with the edge lengths L_x = 15 cm and L_y = 150 cm is to be marked. However, in contrast to chap. J.1.3, **the same** ruler is used for the x-direction and the y-direction.

Input quantities

See chapter J.1.3

Model

See chapter J.1.3

Standard uncertainties of the input quantities

See chapter J.1.3

In addition, the angle-independent uncertainty contribution as determined in chap. J.1.2 is needed:

$$u_{Ly} = u_L = 0.7\ \text{mm}$$

Standard uncertainty of the output quantity

Since the measurements in the x-direction and the y-direction are performed using **the same** ruler, so that both measurement results can be influenced by the ruler in the same way, a correlation term has to be considered. It should be noted that only the length uncertainties in the x-direction the and y-direction have to be included in the correlation. The angle uncertainty, however, must not be included since uncertainties due to angular deviations between ruler elements cannot occur in the x-direction (short side). Accordingly, the basic equation of chap. J.1.3 expanded by a correlation term (3rd summand under the root symbol) applies:

$$\frac{u_C}{A} = \sqrt{\left(\frac{u_x}{L_x}\right)^2 + \left(\frac{u_y}{L_y}\right)^2 + 2\cdot\left(\frac{u_x}{L_x}\right)\cdot\left(\frac{u_{Ly}}{L_y}\right)}$$

$$\approx \sqrt{\left(\frac{0.5\ \text{mm}}{150\ \text{mm}}\right)^2 + \left(\frac{0.877\ \text{mm}}{1500\ \text{mm}}\right)^2 + 2\cdot\left(\frac{0.5\ \text{mm}}{150\ \text{mm}}\right)\cdot\left(\frac{0.7\ \text{mm}}{1500\ \text{mm}}\right)} \approx 0.003816$$

Thus the standard uncertainty u_C of the area

$$A = L_x \cdot L_y = 150\ \text{mm} \cdot 1500\ \text{mm} = 225000\ \text{mm}^2 = 2250\ \text{cm}^2$$

is

$$u_C = 0.003816 \cdot 225000\ \text{mm}^2 = 858.6\ \text{mm}^2 \approx 8.6\ \text{cm}^2$$

Expanded measurement uncertainty

The expanded measurement uncertainty is calculated using $k_p = 2$:

$$U = k_p \cdot u_C = 2 \cdot 858.6\ \text{mm}^2 = 1717.2\ \text{mm}^2 \approx 17.2\ \text{cm}^2$$

Complete measurement result

$$A \pm U = (2250.0 \pm 17.2)\ \text{cm}^2$$

Accordingly, in case of marking a rectangular area of nominal size A = 2250 cm^2, the actual size of the marked area ranges between A = 2232.8 cm^2 and A = 2267.2 cm^2 with a confidence interval of 95.45% (according to $k_p = 2$). These are approximately 0.76% uncertainty in relation to the nominal size.

J.2 Evaluating the suitability of a dial gauge

Description of the measurement

A dial gauge is to be calibrated for the special use case of testing a product characteristic on compliance with the specification (8.0 ± 0.1) mm (T = 200 µm).

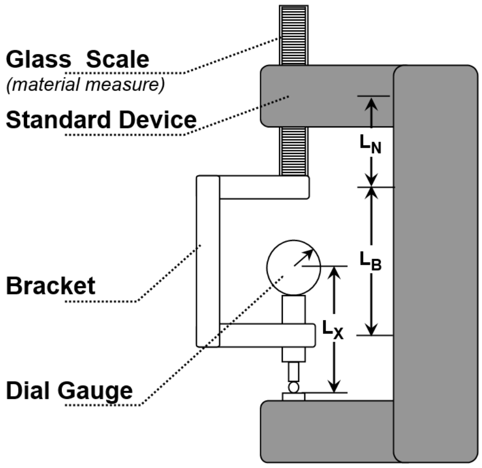

Figure 14: Calibrating a dial gauge

NOTE: With the aid of the bracket the dial gauge is "adapted" to the standard device and thus the calibration is enabled.

Input quantities

- **Information about the standard device**

  Manufacturer's specification of the measurement uncertainty: $U_{CAL} = 0.4\,\mu m + 0.6 \cdot 10^{-6} \cdot L_I$
  L_I – indicated length in µm, $k_p = 2$, temperature range (20 ± 0.5) °C

  Digit increment of the indication $\Delta L_I = 0.1\,\mu m$

- **Information about the object to be measured** Dial gauge as per ISO 463

  Scale interval $SI = 0.01\,mm$

  Uncertainty of the estimate of the pointer position on the scale $\Delta SI = 0.1 \cdot SI$

  Length of the measuring bolt $L_X = 100\ mm$

  Linear thermal coefficient of expansion of the measuring bolt $\alpha_X = (8.5 \pm 1.5) \cdot 10^{-6}\ K^{-1}$

- **Information about the procedure**

  Temperature deviation from $\vartheta_0 = 20\,°C$ during measurement $\Delta\vartheta = 1\,K$

  Length of the bracket $L_B = 200\ mm$

  Linear thermal expansion coefficient of the bracket $\alpha_B = (10.5 \pm 1.5) \cdot 10^{-6}\ K^{-1}$

  Effective length of the glass scale of the standard device $L_N = 70\ mm$

  Linear thermal expansion coefficient of the glass scale $\alpha_N = (11.5 \pm 1.5) \cdot 10^{-6}\ K^{-1}$

  NOTE: It is assumed that the solid parts of the standard device do not change during the short period of measurement time as a result of temperature fluctuations in the $\pm\Delta\vartheta$ range.

Model

$$y = \underbrace{y' + K}_{= y_0} + \delta x_{CAL} + \delta x_O + \delta x_N + \delta x_X + \delta x_B$$

with

y	Indication of the dial gauge,
y'	Uncorrected indication of the dial gauge,
K	Correction,
y_0	Indication of the standard device (conventional value, no uncertainty),
δx_{CAL}	Deviation due to the limited precision of standard device calibration,
δx_O	Deviation due to the limited accuracy of the scale readability,
δx_N	Deviation due to the temperature influence on the standard device,
δx_X	Deviation due to the temperature influence on the measuring object,
δx_B	Deviation due to the temperature influence on the bracket.

$-\Delta x \le \delta x \le \Delta x$ applies to all above-mentioned deviations. Here, δx describes the instantaneous value of the fluctuating deviation (expected value $\delta x = 0$), Δx the associated maximum deviation.

Measurement results

Measured displacement between the pointer positions 0 mm (initial position) and 8 mm (end position): When the pointer position is $y' = 8.00\ \text{mm}$, the standard device indicates the measured displacement $y_0 = 8.022\ \text{mm}$.

Correction

The deviation of the dial gauge indication y' from the conventional value y_0 of the standard device is $-22\ \mu\text{m}$, i.e.

$$K = y_0 - y' = 8{,}022\ \text{mm} - 8{,}00\ \text{mm} = 0{,}022\ \text{mm} = 22\ \mu\text{m}$$

NOTE: This correction applies ***exclusively*** *to the dial gauge indication y' = 8 mm. In order to calibrate the entire measuring range of the dial gauge, measurements at various indications (calibration points) distributed throughout the measuring range and evaluation according to appendix F are required. This often leads to corrections that are dependent on the respective displacement and additional uncertainties.*

In practice, corrections are not common for this type of dial gauge so that the systematic error must be considered as an uncertainty contribution in the uncertainty budget (see appendix F.3).

Standard uncertainties of the input quantities

- **Standard device**: Standard uncertainty in case of the measured displacement $L_I = y_0 = 8{,}022\ \text{mm}$ and assuming a normal distribution

  $$u_{CAL} = \frac{U_{CAL}}{k_p} = \frac{0.4\ \mu\text{m} + 0.6 \cdot 10^{-6} \cdot 8022\ \mu\text{m}}{2} = \frac{0.4\ \mu\text{m} + 0.0048\ \mu\text{m}}{2} = 0.2024\ \mu\text{m} \approx 0.203\ \mu\text{m}$$

  The standard uncertainty of the digit increment is included in this uncertainty.

- **Measuring object**: Standard uncertainty due to the uncertainty of the scale reading

  Upper and lower limit values for the deviation of the reading value from the pointer position:

  $$a_+ = +\Delta SI = +0.1 \cdot SI = +0.1 \cdot 0{,}01\ \text{mm} = +1.0\ \mu\text{m}$$

  $$a_- = -\Delta SI = -0.1 \cdot SI = -0.1 \cdot 0.01\ \text{mm} = -1.0\ \mu\text{m}$$

  Standard uncertainty assuming rectangular distribution:

  $$u_O = \frac{a}{\sqrt{3}} = \frac{a_+ - a_-}{2} \frac{1}{\sqrt{3}} = \frac{1.0\ \mu\text{m}}{\sqrt{3}} \approx 0.5774\ \mu\text{m} \approx 0.578\ \mu\text{m}$$

- **Procedure:** Standard uncertainty of the (effective) glass scale length L_N of the **standard device** due to deviations of the ambient temperature from the reference temperature $\vartheta_0 = 20\ °C$

  Upper and lower limit values of the deviations from L_N:

  $a_+ = \alpha_N \cdot L_N \cdot (+\Delta\vartheta) = 11.5 \cdot 10^{-6}\ K^{-1} \cdot 70\ mm \cdot (+1K) = 0.000805\ mm = 0.805\ \mu m$

  $a_- = \alpha_N \cdot L_N \cdot (-\Delta\vartheta) = 11.5 \cdot 10^{-6}\ K^{-1} \cdot 70\ mm \cdot (-1K) = -0.000805\ mm = -0.805\ \mu m$

  Standard uncertainty assuming rectangular distribution:

  $$u_N = \frac{a}{\sqrt{3}} = \frac{a_+ - a_-}{2}\frac{1}{\sqrt{3}} = \frac{0.805\ \mu m}{\sqrt{3}} \approx 0.465\ \mu m$$

- **Procedure:** Standard uncertainty of the **measuring bolt length** L_X of the measuring instrument due to deviations of the ambient temperature from the reference temperature $\vartheta_0 = 20\ °C$

  Upper and lower limit values of the deviations from L_X:

  $a_+ = \alpha_X \cdot L_X \cdot (+\Delta\vartheta) = 8.5 \cdot 10^{-6}\ K^{-1} \cdot 100\ mm \cdot (+1K) = 0.00085\ mm = 0.85\ \mu m$

  $a_- = \alpha_X \cdot L_X \cdot (-\Delta\vartheta) = 8.5 \cdot 10^{-6}\ K^{-1} \cdot 100\ mm \cdot (-1K) = -0.00085\ mm = -0.85\ \mu m$

  Standard uncertainty assuming rectangular distribution:

  $$u_X = \frac{a}{\sqrt{3}} = \frac{a_+ - a_-}{2}\frac{1}{\sqrt{3}} = \frac{0.85\ \mu m}{\sqrt{3}} \approx 0.491\ \mu m$$

- **Procedure:** Standard uncertainty of the **bracket length** L_B due to deviations of the ambient temperature from the reference temperature $\vartheta_0 = 20\ °C$

  Upper and lower limit values of the deviations from L_B:

  $a_+ = \alpha_B \cdot L_B \cdot (+\Delta\vartheta) = 10.5 \cdot 10^{-6}\ K^{-1} \cdot 200\ mm \cdot (+1K) = 0.00210\ mm = 2.10\ \mu m$

  $a_- = \alpha_B \cdot L_B \cdot (-\Delta\vartheta) = 10.5 \cdot 10^{-6}\ K^{-1} \cdot 200\ mm \cdot (-1K) = -0.00210\ mm = -2.10\ \mu m$

  Standard uncertainty assuming rectangular distribution:

  $$u_B = \frac{a}{\sqrt{3}} = \frac{a_+ - a_-}{2}\frac{1}{\sqrt{3}} = \frac{2.10\ \mu m}{\sqrt{3}} \approx 1.2124\ \mu m \approx 1.213\ \mu m$$

Standard uncertainty of the output quantity

$$u_C = \sqrt{u_{CAL}{}^2 + u_O{}^2 + u_N{}^2 + u_X{}^2 + u_B{}^2 + K^2}$$
$$= \sqrt{(0.203^2 + 0.578^2 + 0.465^2 + 0.491^2 + 1.213^2 + 22^2)\mu m^2} \approx \sqrt{486.304}\ \mu m \approx 22.053\ \mu m$$

Expanded measurement uncertainty

With the coverage factor $k_p = 2$ the expanded measurement uncertainty of the calibration results in

$U = k_p \cdot u_C \approx 2 \cdot 22.053\ \mu m = 44.106\ \mu m \approx 44.2\ \mu m$

Complete measurement result

$y = y' \pm U = (8000 \pm 44.2)\ \mu m = 8.0\ mm \pm 44.2\ \mu m$

Accordingly, the conventional value of the measurement result can be expected in the range between 7.955 mm and 8.045 mm with a confidence level of 95.45%. This applies to the 8 mm measuring point only.

Conclusion: $U/T = 44{,}2\ \mu m / 200\ \mu m > 0{,}22$ (22%) violates the "golden rule of metrology" according to which U/T preferably should be less than 10%, but not at all greater than 20%. Therefore, it does not make sense to use the dial gauge for the intended task (see chapter 2.2, note 1).

NOTE: Correcting the indications could reduce the uncertainty to U < 3.1 µm so that U/T < 0.02 (2%).

Information about input quantities							Standard uncertainties of input quantities				Contributions to the measurement uncertainty of the measurand				
Seq. No.	Description	Variable (symbol)	Measuring unit	Value of the variable	Value of the uncertainty data	Comments (z.B. references, explanatory notes, links to documents)	Evaluation type	Type A: Number of measured values; Type B: kp (≥1), confidence level (%), distribution	Numerical factor for calculating the standard uncertainty	Standard uncertainty	Sensitivity coefficient	Contribution to uncertainty	Contribution to uncertainty (squared)	Percentage contribution to MU budget $\frac{(c_i \cdot u(x_i))^2}{\sum_{i=1}^{n}(c_i \cdot u(x_i))^2}$	Rank (according to Pareto)
i				x_i	Δx_i		A B	m_i k_p, %, name	1 or $\sqrt{m_i}$ k_p	$u(x_i) = \Delta x_i / k_p$	c_i	$c_i * u(x_i)$	$(c_i * u(x_i))^2$	[%]	
1	Standard: Calibration	δx_{CAL}	µm	0	0,405	See text for calculation	B	Normal distribution (95%)	2,000	0,203	1	0,203	0,041209	0,0%	6
2	Measuring object: Reading of scale	δx_O	µm	0	1,000	See text for calculation	B	Rectangular distribution	1,732	0,578	1	0,578	0,334084	0,0687%	3
3	Procedure: Length of glass scale	δx_N	µm	0	0,805	See text for calculation	B	Rectangular distribution	1,732	0,465	1	0,465	0,216225	0,0%	5
4	Procedure: Length of measuring bolt	δx_x	µm	0	0,850	See text for calculation	B	Rectangular distribution	1,732	0,491	1	0,491	0,241081	0,0496%	4
5	Procedure: Length of bracket	δx_B	µm	0	2,100	See text for calculation	B	Rectangular distribution	1,732	1,213	1	1,213	1,471369	0,3026%	2
6	Systematic measurement error	$y' - y_0$	µm	-22	22,000	Basis: 1 measured value *(no statistical evaluation)*	B	1	1,000	22,000	1	22,000	484,000000	99,5%	1
7															
8															
9															
10															
11															
Model equation:							**Total result:**					**u_c^2 =**	486,304	100,000%	
												u_c =	22,053		
												k_p =	2,000		
												U =	**44,106**		

$$y = \underbrace{y' + K}_{= y_0} + \delta x_{CAL} + \delta x_O + \delta x_N + \delta x_X + \delta x_B$$

Expected values: $\delta x = 0$ Deviations: $-\Delta x \le \delta x \le \Delta x$

Table 8: Uncertainty budget for the “dial gauge” example

J.3 Measuring a bolt diameter

The example shows the basic procedure for determining the uncertainty of a measurement result according to [GUM]. This includes determining the correction, the correction uncertainty and the coverage factor k_p for the expanded measurement uncertainty of the output quantity using the degrees of freedom. Only a few less significant uncertainties are disregarded right from the beginning (e.g. uncertainties of the thermal coefficient of expansion). By means of the evaluation, the input quantities with major and minor impact on the measurement uncertainty can be distinguished.

NOTE 1: In operational practice, it is customary to neglect input quantities that are classified (after thorough examination) as being less significant or not at all significant.

NOTE 2: Roundings must be performed in line with the rules according to chap. 4.7.2. If the evaluation is largely done without roundings of intermediate results, smaller values for the measurement uncertainty of the output quantity (diameter) may result.

Description of the measurement

The diameter of a bolt is measured using a comparator with the measuring object being inserted between two plane-parallel measuring surfaces (probing planes):

- Two-point measurement between plane surfaces, fully float-mounted,
- Measurements at m = 8 different points of the bolt circumference,
- Temperatures of the measuring object and the glass scale are measured.

Zero compensation is performed prior to measuring.

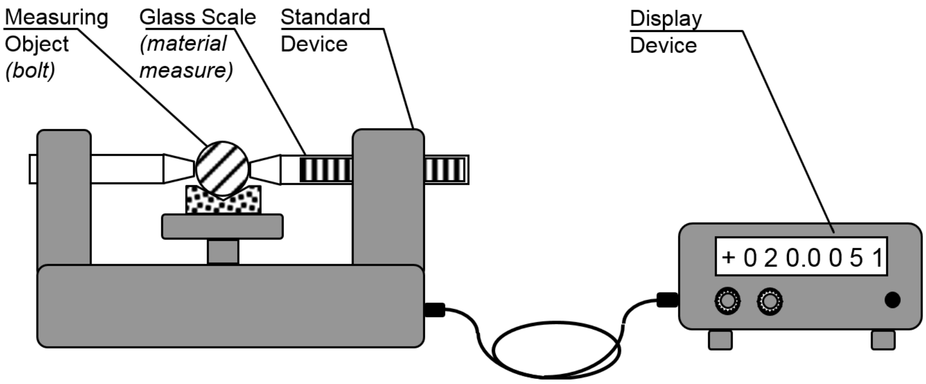

Figure 15: Measuring setup for the measurement of a diameter

Input quantities

- Standard device:

  Manufacturer's specification of the measurement uncertainty: $U_N = 0.3\,\mu m + 1 \cdot 10^{-6} \cdot L_I$
  L_I – indicated length in µm, k_p = 2, temperature range (20 ± 0.5) °C

  Thermal coefficient of expansion (glass scale): $\alpha_N = 8 \cdot 10^{-6} \cdot K^{-1}$

  Digit increment of the indication: $\Delta L_I = 0{,}1\,\mu m$

- Measuring object:

  Nominal diameter of the bolt (at $\vartheta_0 = 20\,°C$): $L_O = 20\,mm = 20000\,\mu m$

  Thermal coefficient of expansion (aluminum): $\alpha_O = 24 \cdot 10^{-6}\,K^{-1}$

- Measurement procedure:

  Number of different measuring points: $m = 8$

  Uncertainty of the alignment of the measuring object: $U_A = 0.15\,\mu m$
  $k_p = 2$; U_A known from m = 25 previous measurements under the same conditions

  Uncertainty of the probing operation due to deviations of the probing planes from plane parallelism: $U_P = 0.15\,\mu m$
  $k_p = 2$; U_P known from previous measurements under the same conditions with a standard

  Temperature of the glass scale during the measurement: $\vartheta_N = 23.5\,°C$

  Temperature of the measuring object during the measurement: $\vartheta_O = 25.0\,°C$

  Uncertainty of the thermometer: $U_\vartheta = 0.5\,K$
  Thermometer with resolution of 0.1 K

<u>Model</u>

$$y = \underbrace{y' + K}_{= y_0} + \delta x_N + \delta x_R + \delta x_A + \delta x_P + \delta K$$

with

y	Indication for the diameter,
y'	Uncorrected indication,
K	Correction,
y_0	Corrected indication (conventional value, no uncertainty),
δx_N	Deviation due to the limited precision of standard device calibration,
δx_R	Deviation due to the dispersion during repeated measurements,
δx_A	Deviation due to the inaccurate alignment of the measuring object,
δx_P	Deviation due to inexact plane-parallel probing planes,
δK	Deviation due to inaccurate correction of the systematic measurement error resulting from limited temperature measurement accuracy.

$-\Delta x \le \delta x \le \Delta x$ applies to all above-mentioned deviations. Here, δx describes the instantaneous value of the fluctuating deviation (expected value $\delta x = 0$), Δx the associated maximum deviation.

<u>Measurement results</u>

Measurement no.	1	2	3	4	5	6	7	8
∅ in mm	20.0052	20.0045	20.0055	20.0047	20.0051	20.0046	20.0053	20.0051

Mean value: $\bar{x} = 20.0050\,mm$
Standard deviation: $s = 0.00036\,mm = 0.36\,\mu m$
The mean value is considered to be an uncorrected measurement result: $y' = \bar{x}$

Correction

At operating temperatures that deviate from the reference temperature of 20 °C, systematic errors may occur due to different changes in the lengths of the measuring system components and of the measuring object. In the present case, it is assumed that the relevant changes in length of the glass scale and the solid parts of the standard device neutralize each other except for insignificant proportions so that only the measuring object must be considered.

NOTE: This assumption is possibly no longer justified for operating temperatures that deviate significantly from the reference temperature. In this case, temperature influences on the standard device must be taken into account as well. Accordingly, the determination of the correction will be more complex.

Thermal expansion of the measuring object:
$\Delta L_O = \alpha_O \cdot (\vartheta_O - 20\ °C) \cdot L_O = 24 \cdot 10^{-6}\ K^{-1} \cdot (25 - 20)\ K \cdot 20000\ \mu m = 2.4\ \mu m$

Correction of the bold diameter:
$K = -\Delta L_O = -2.4\ \mu m = -0.0024\ mm$

Corrected measurement result according to appendix F:
$y_0 = y' + K = 20.0050\ mm + (-0.0024\ mm) = 20.0026\ mm$

Standard uncertainties of the input quantities

- **Standard device**: The standard uncertainty for a measured displacement of $L_N = 20\ mm$ is determined using the calculation rule for the measurement uncertainty specified by the manufacturer:

  $U_N = 0.3\ \mu m + 1 \cdot 10^{-6} \cdot L_N = 0.3\ \mu m + 1 \cdot 10^{-6} \cdot 20000\ \mu m = (0.3 + 0.02)\ \mu m = 0.32\ \mu m$

  For this expanded measurement uncertainty, a normal distribution with a confidence interval of 95.45% is assumed, i.e. $k_p = 2$. Standard uncertainty resulting from $k_p = 2$:

  $u_N = \frac{U_N}{2} = \frac{0.32\ \mu m}{2} = 0.16\ \mu m$

  Degrees of freedom according to chap. 4.4.2.1:
  $\nu_N \to \infty$

- **Standard device**: The standard uncertainty due to the digit increment of the indication is included in the measurement uncertainty specified by the manufacturer and in the measurement series dispersion.

- **Measuring object**: The measuring object does not contribute to the uncertainty budget, since the measurements are performed at eight different points on the measuring object so that the effect of shape deviations is included in great part in the measured values of repeated measurements.

- **Procedure**: Standard uncertainty due to repeated measurements on the measuring object

  The measurement results of repeated measurements are considered to be normally distributed. Standard uncertainty according to chap. 4.4.1.1:

  $u_R = \frac{s}{\sqrt{m}} = \frac{0.36\ \mu m}{\sqrt{8}} \approx 0.13\ \mu m$

  Degrees of freedom according to appendix D.3.1:
  $\nu_R = m - 1 = 8 - 1 = 7$

- **Procedure**: Standard uncertainty due to inexact alignment of the measuring object

  The empirical value $U_A = 0{,}15\ \mu m$ with a confidence interval of 95.45% is available from previous measurements for the alignment uncertainty. This uncertainty was determined based on m = 25 repeated measurements.

Degrees of freedom according to appendix D.3.1:

$$\nu_A = m - 1 = 25 - 1 = 24$$

According to appendix D.1 the coverage factor $k_p \approx 2$ results in case of ν = 24 degrees of freedom and a confidence interval of 95.45%. Standard uncertainty according to chap. 4.4.2.1:

$$u_A = \frac{U_A}{k_p} = \frac{0.15\,\mu m}{2} \approx 0.08\,\mu m$$

- **Procedure**: Standard uncertainty due to inexactly plane-parallel probing planes

  For the probing uncertainty due to probing planes that are not plane-parallel, the empirical value $U_P = 0{,}15\,\mu m$ with a confidence level of 95.45% is available. Standard uncertainty according to chap. 4.4.2.1:

  $$u_P = \frac{U_P}{k_p} = \frac{0.15\,\mu m}{2} \approx 0.08\,\mu m$$

  Degrees of freedom according to chap. 4.4.2.1:

  $$\nu_P \to \infty$$

- **Procedure**: Standard uncertainty of the correction due to temperature measurement uncertainty

  The uncertainty $U_\vartheta = 0{,}5\,K$ of the thermometer results in the following limit values [25]:

  $$L_O^{(+)} = L_O + \alpha_O \cdot (\vartheta_O + U_\vartheta - 20\,°C) \cdot L_O = 24 \cdot 10^{-6}\,K^{-1} \cdot (25 + 0.5 - 20)\,K \cdot 20000\,\mu m \approx 2.64\,\mu m$$

  $$L_O^{(-)} = L_O + \alpha_O \cdot (\vartheta_O - U_\vartheta - 20\,°C) \cdot L_O = 24 \cdot 10^{-6}\,K^{-1} \cdot (25 - 0.5 - 20)\,K \cdot 20000\,\mu m \approx 2.16\,\mu m$$

  Standard uncertainty according to chap. 4.4.2.2 assuming rectangular distribution:

  $$u_K = \frac{a}{\sqrt{3}} = \frac{L_O^{(+)} - L_O^{(-)}}{2} \cdot \frac{1}{\sqrt{3}} \approx \frac{2.64\,\mu m - 2.16\,\mu m}{2} \cdot \frac{1}{1.732} \approx 0.14\,\mu m$$

  The uncertainty $U_\vartheta = 0.5\,K$ of the temperature recording is estimated to be uncertain at 50%. Then, an uncertainty of 50% also results for u_K. Corresponding degrees of freedom according to appendix D.3.2:

  $$\nu_K \approx \frac{1}{2}\left(\frac{\Delta u_K}{u_K}\right)^{-2} = \frac{1}{2}(0.5)^{-2} = \frac{1}{2 \cdot 0.5^2} = 2$$

<u>Standard uncertainty of the output quantity</u>

Combined standard uncertainty according to chap. 4.5:

$$u_C = \sqrt{{u_N}^2 + {u_R}^2 + {u_A}^2 + {u_P}^2 + {u_K}^2}$$
$$= \sqrt{(0.16\,\mu m)^2 + (0.13\,\mu m)^2 + (0.08\,\mu m)^2 + (0.08\,\mu m)^2 + (0.14\,\mu m)^2} \approx 0.2737\,\mu m \approx 0.28\,\mu m$$

Degrees of freedom according to appendix D.3.3 (Welch-Satterthwaite equation):

$$\nu_{eff} = \frac{{u_C}^4}{\frac{{u_N}^4}{\nu_N} + \frac{{u_R}^4}{\nu_R} + \frac{{u_A}^4}{\nu_A} + \frac{{u_P}^4}{\nu_P} + \frac{{u_K}^4}{\nu_K}}$$
$$= \frac{(0.28\,\mu m)^4}{\lim\limits_{\nu_N \to \infty} \frac{(0.16\,\mu m)^4}{\nu_N} + \frac{(0.13\,\mu m)^4}{7} + \frac{(0.08\,\mu m)^4}{24} + \lim\limits_{\nu_P \to \infty} \frac{(0.08\,\mu m)^4}{\nu_P} + \frac{(0.14\,\mu m)^4}{2}}$$
$$= \frac{0.006147}{0 + 0.000041 + 0.000002 + 0 + 0.000192} = \frac{0.006147}{0.000235} \approx 26.1574 \approx 26$$

25 See note 5 on page 111 on the subject of using the expanded measurement uncertainty U_ϑ as a deviation $\Delta\vartheta$

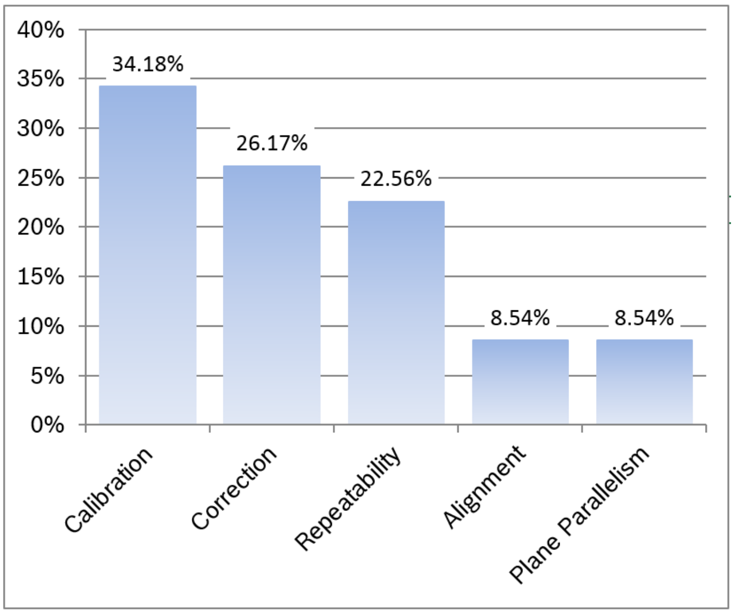

Figure 16: Bolt diameter; Pareto chart of the uncertainty contributions u_i^2

On the basis of the chart, a reduction of the measurement uncertainty up to approximately 25% can be expected if, for example, the uncertainty of the correction could be reduced. If applicable, it should be checked whether a reduction can be achieved by means of an improved adjustment of the operating temperature to the reference temperature 20°C and the associated smaller correction.

Expanded measurement uncertainty

According to appendix D.1, $\nu_{eff} = 26$ degrees of freedom together with a confidence level of 95.45% result in the coverage factor $k_p = 2.10$.

Expanded measurement uncertainty according to chap. 4.6:

$$U = k_p \cdot u_C = 2.10 \cdot 0.28\ \mu m = 0.59\ \mu m \approx 0.6\ \mu m$$

NOTE: Without an analysis of the degrees of freedom, typically the coverage factor k_p = 2.00 is used. In doing so, it is (often tacitly and in a manner that is not always justified) assumed that $\nu \geq 20$ degrees of freedom are present. This leads to the slightly lower measurement uncertainty of U = 0.56 µm. Rounded up to the nearest decimal place (see chap. 4.7.2), however, the result is also U = 0.6 µm.

This expanded measurement uncertainty – which is calculated taking account of the boundary conditions described above – is only valid for the period of time when the measurement is carried out. If the uncertainty shall be also valid for later measurements, influencing quantities which might additionally take effect during this period of time must be considered as well.

Complete measurement result

Complete measurement result according to chap. 4.7:

$$y = y_0 \pm U = y' + K \pm U = (20005.0 - 2.4 \pm 0.6)\ \mu m = 20.0026\ mm \pm 0.6\ \mu m$$

The conventional value of the measurement result can be expected in the range between 20.0020 mm and 20.0032 mm with a confidence level of 95.45%.

Information about input quantities							Standard uncertainties of input quantities				Contributions to the measurement uncertainty of the measurand					Determining kp for the measurand		
Seq. No.	Description	Variable (symbol)	Measuring unit	Value of the variable	Value of the uncertainty data	Comments (z.B. references, explanatory notes, links to documents)	Evaluation type	Type A: Number of measured values; Type B: kp (≥1), confidence level (%), distribution	Numerical factor for calculating the standard uncertainty	Standard uncertainty	Sensitivity coefficient	Contribution to uncertainty	Contribution to uncertainty (squared)	Percentage contribution to MU budget $\frac{(c_i \cdot u(x_i))^2}{\sum_{i=1}^{n}(c_i \cdot u(x_i))^2}$	Rank (according to Pareto)	Estimated uncertainty of the uncertainty data	Degrees of freedom	Contribution to the denominator of the Welch-Satterthwaite formula
i				x_i	Δx_i		A B	m_i k_p, %, name	1 or $\frac{\sqrt{m_i}}{k_p}$	$u(x_i) = \Delta x_i / k_p$	c_i	$c_i * u(x_i)$	$(c_i * u(x_i))^2$	[%]		$\Delta u(x_i) / u(x_i)$	ν_i	$\frac{(c_i \cdot u(x_i))^4}{\nu_i}$
1	Standard: Calibration (travel)	δx_N	µm	0	0,32	See text for calculation (using manufacturer's calculation rule)	B	Normal distribution (95%)	2,000	0,16	1	0,16	0,0256	34,179%	1		1E+99	0,000000
2	Standard: Digit increment	Contained in the manufacturer's uncertainty data (δx_N) and in the dispersion of the measurement series (δx_R)																
3	Measuring object	Measurement at eight different measuring points, shape deviations contained in the dispersion of the measurement series (δx_R)																
4	Procedure: Repeatability	δx_R	µm	0	0,36	Measurement	A	8,000	2,828	0,13	1	0,13	0,0169	22,563%	3		7	0,000041
5	Procedure: Alignment	δx_A	µm	0	0,15	Experience	B	Normal distribution (95%)	2,000	0,08	1	0,08	0,0064	8,545%	4		24	0,000002
6	Procedure: Plane Parallelism	δx_P	µm	0	0,15	Experience	B	Normal distribution (95%)	2,000	0,08	1	0,08	0,0064	8,545%	5		1E+99	0,000000
7	Procedure: Correction (temperature)	δK	µm	0	0,14	See text for calculation	B	1,000	1,000	0,14	1	0,14	0,0196	26,168%	2	50,0%	2	0,000192
8																		
9																		
10																		
11																		
							Total result:					u_c^2 =	0,0749	100,000%			$\sum_i$ =	0,000235
												u_c =	0,28				ν_{eff} =	26
												k_p =	2,10				$1 - \alpha$ =	95,45%
												U =	**0,59**				k_p =	2,10

Model equation:

$$y = \underbrace{y' + K}_{= y_0} + \delta x_N + \delta x_R + \delta x_A + \delta x_P + \delta K$$

Expected values: $\delta x = 0$ Deviations: $-\Delta x \le \delta x \le \Delta x$

Table 9: Uncertainty budget for the "bolt diameter" example

J.4 Torque measurement using an engine test station

<u>Description of the measurement</u>

Engine test stations include torque measuring equipment. Figure 17 presents the measurement chain schematically. The measurement tasks vary greatly across different test stations and instants of time. Thus, it is impossible to determine the uncertainty of the measurement results for each individual case. Instead, it is determined once for certain reference values and then applied to all structurally identical systems and measurements that are performed under the same conditions. The approach is explained by means of the reference point M_0 = 100 Nm as an example. The same procedure is used for other reference points.

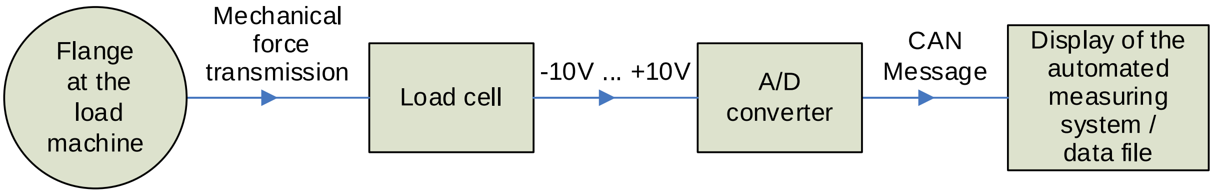

Figure 17: Measuring chain of an engine test station, typical measuring range: –50 Nm to +500 Nm

Using the engine test station, the torque is determined that acts on the flange between the engine and the load machine. The load machine is simultaneously used as a measuring instrument and provides a load cell for this purpose. The torque is calculated from the measured force and the known length of the lever arm of the mechanical system.

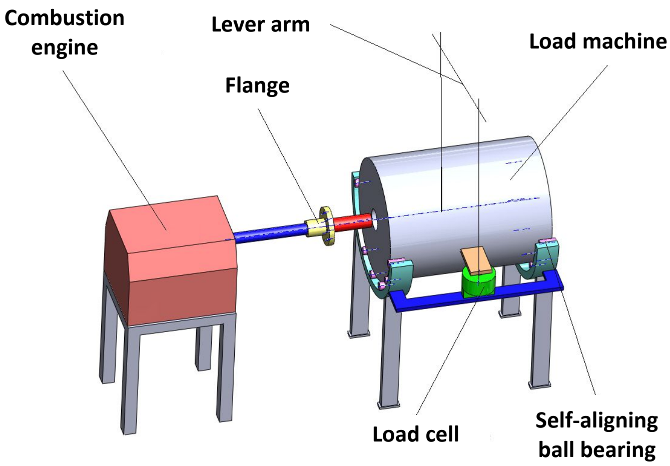

Figure 18: Schematic structure of an engine test station

Recurrent calibration of the entire measuring chain is essential for the practical use of the measuring process (Figure 17). For this purpose, the engine is replaced with a torque measurement standard at the connection flange which is traced back to national and international primary standards (calibration certificate). The standard essentially consists of a mechanical lever arm and calibrated reference masses [26] exerting defined reference forces on the load cell. Depending on the calibration result, the system is adjusted and re-calibrated as required.

<u>Input quantities</u>

- **Torque** (reference value) $M_0 = 100\ \text{Nm}$
- **Resolution of indication** (digit increment) $\Delta M_R = 0.05\ \text{Nm}$
- **Lever arm length, nominal value** (manufacturer's specification) $L_0 = 1000\ \text{mm}$
- **Maximum deviation of the lever arm length** from the nominal value (based on manufacturer's specifications) $\Delta L = 0.32\ \text{mm}$

[26] Also imprecisely described as “weight piece” (see DIN 8127:2007-11) or coll. “weight”

- **Maximum deviation of the reference masses** from the nominal value (manufacturer's specifications) — $\%\Delta_m = 0.005\%$
- **Ambient temperature** during calibration — $\vartheta_0 = 20.0\ °C$
- **Maximum deviation of the ambient temperature** during calibration — $\Delta\vartheta_0 = 3.0\ K$
- **Maximum deviation of the ambient temperature** during measurement — $\Delta\vartheta = 6.0\ K$
- **Maximum deviation of the torque indication** due to deviation of the load cell resulting from temperature deviation (manufacturer's specifications) — $\%\Delta_\vartheta = 0.05\%/K$ *(related to M_0)*
- **Full scale value of the measuring range** — $M_{MAX} = 500\ Nm$
- **Maximum permissible deviation** between reference value and indication within which the measuring system is classified as OK when calibrated — $\%\Delta = 0{,}4\%$ *(based on M_{MAX})*

  This "acceptance range" $\%\Delta$ is used to consider the effect of the following effects:
  - The torque which is actually effective at the flange is only indirectly recorded via the load cell and the lever arm length.
  - Friction in the bearings of the lever arm leads to measurement errors and hysteresis of the calibration curve.
  - The zero point and sensitivity of the entire system have a long-term drift.

  These effects are not compensated by recurring adjustment and calibration. Instead, the control of inspection, measuring and test equipment is utilized to ensure that the overall impact of these effects remains within specified limits (± 0.4% of the full scale value of the measuring range).

<u>Model equation</u>

$$M = M_0 + \delta M_R + \delta M_L + \delta M_m + \delta M_\vartheta + \delta M_\Delta$$

with

M	Indication for the torque,
M_0	Conventional value (no uncertainty),
δM_R	Deviation due to the limited resolution of the measuring system,
δM_L	Deviation due to the uncertainty of the lever arm length,
δM_m	Deviation due to the uncertainty of the reference masses,
δM_ϑ	Deviation due to the uncertainty of the force measurement resulting from temperature fluctuation,
δM_Δ	Deviation due to the uncertainty of the difference between the reference value and the indication.

$-\Delta M \le \delta M \le \Delta M$ applies to all above-mentioned deviations. Here, δM describes instantaneous value of the fluctuating deviation (expected value $\delta M = 0$), ΔM the associated maximum deviation.

<u>Measurement results</u>

No measurements are carried out, all details are taken from the manufacturer's data sheets or they are based on experience.

<u>Correction</u>

No corrections are performed.

Standard uncertainties of the input quantities

- The limited **resolution** ΔM_R = 0.05 Nm (digit increment) of the torque indication can lead to deviations within the limits

  $a_+ = +\frac{\Delta M_R}{2}$ and $a_- = -a_+ = -\frac{\Delta M_R}{2}$

  i.e. cause the maximum deviation

  $$a = \frac{a_+ - a_-}{2} = a_+ = \frac{\Delta M_R}{2} = 0.025\,\text{Nm}$$

  Assuming a rectangular distribution results in the standard uncertainty

  $$u_R = \frac{a}{\sqrt{3}} = \frac{0.025}{\sqrt{3}}\,\text{Nm} \approx 0.015\,\text{Nm}$$

- The **lever arm length** is uncertain during calibration with respect to the manufacturing tolerance of the lever arm and its mechanical mounting. Moreover, temperature fluctuations up to $\delta\vartheta_0 = \pm\Delta\vartheta_0 = \pm 3\,\text{K}$ are assumed which may occur during the calibration process without corrections being made. These effects all in all can cause deviations of the lever arm length up to $\delta L = \pm\Delta L = \pm 0{,}32\,\text{mm}$ (which is determined based on manufacturer's specifications). Furthermore it is assumed that torque and lever arm length change with the same ratio, i.e. proportionally:

  $$\frac{\delta M_L}{M_0} = \frac{\delta L}{L_0}$$

  This may lead to deviations of the measured torque within the limits

  $a_+ = +\Delta M_L = \frac{\Delta L}{L_0} \cdot M_0$ and $a_- = -a_+ = -\Delta M_L = -\frac{\Delta L}{L_0} \cdot M_0$

  i.e. cause the maximum deviation

  $$a = \frac{a_+ - a_-}{2} = \frac{\Delta L}{L_0} \cdot M_0 = \frac{0.32\,\text{mm}}{1000\,\text{mm}} \cdot 100\,\text{Nm} = 0.032\,\text{Nm}$$

  Assuming a rectangular distribution results in the standard uncertainty

  $$u_L = \frac{a}{\sqrt{3}} = \frac{0.032}{\sqrt{3}}\,\text{Nm} \approx 0.019\,\text{Nm}$$

- For the **reference masses** the tolerances $\%\Delta_m$ = 0.005% apply which are specified by the manufacturer and related to the nominal value m_0 of the corresponding reference mass. It is assumed that the torque and the reference mass change with the same ratio, i.e. proportionally:

  $$\frac{\delta M_m}{M_0} = \frac{\delta m}{m_0} \leq \frac{\%\Delta_m}{100\%}$$

  This may lead to deviations of the measured torque within the limits

  $a_+ = +\Delta M_m = \frac{\%\Delta_m}{100\%} \cdot M_0$ and $a_- = -a_+ = -\Delta M_m = -\frac{\%\Delta_m}{100\%} \cdot M_0$

  i.e. cause the maximum deviation

  $$a = \frac{a_+ - a_-}{2} = \Delta M_m = \frac{\%\Delta_m}{100\%} \cdot M_0 = \frac{0.005\%}{100\%} \cdot 100\,\text{Nm} = 0.005\,\text{Nm}$$

  Assuming a rectangular distribution results in the standard uncertainty

  $$u_m = \frac{a}{\sqrt{3}} = \frac{0.005}{\sqrt{3}}\,\text{Nm} \approx 0.003\,\text{Nm}$$

  NOTE: The corresponding reference forces $g \cdot m_0$ are calculated using the gravitational acceleration g which is applicable at the operating site of the standard device according to data provided by "Physikalisch-Technische Bundesanstalt" (PTB, Federal Physical-Technical Institute, Germany). Here, however, the uncertainty of g (0.0002%) is evaluated as negligible, so that regardless of whether forces or masses are considered, the same standard uncertainty u_m results.

- The **ambient temperature** during the measurement influences the zero point and the sensitivity of the load cell. In contrast to the calibration procedure (no engine is coupled, so there is no waste heat) temperature fluctuations up to $\delta\vartheta = \pm\Delta\vartheta = \pm 6\,\mathrm{K}$ can occur during measuring operation (engine is coupled, i.e. waste heat is present). Per Kelvin temperature deviation of the load cell from the calibration temperature $\vartheta_0 = 20\,°\mathrm{C}$, a measurement error of $\%\Delta_\vartheta = 0{,}05\%/\mathrm{K}$ from the conventional value M_0 has to be expected (manufacturer's specifications). This may lead to deviations of the measured torque within the limits

  $$a_+ = +\Delta M_\vartheta = \Delta\vartheta \cdot \frac{\%\Delta_\vartheta}{100\%} \cdot M_0 \qquad \text{and} \qquad a_- = -a_+ = -\Delta M_\vartheta = -\Delta\vartheta \cdot \frac{\%\Delta_\vartheta}{100\%} \cdot M_0$$

  i.e. cause the maximum deviation

  $$a = \frac{a_+ - a_-}{2} = \Delta M_\vartheta = \Delta\vartheta \cdot \frac{\%\Delta_\vartheta}{100\%} \cdot M_0 = 6.0\,\mathrm{K} \cdot \frac{0.05\,\frac{\%}{\mathrm{K}}}{100\%} \cdot 100\,\mathrm{Nm} = 0.300\,\mathrm{Nm}$$

  Assuming a rectangular distribution results in the standard uncertainty

  $$u_\vartheta = \frac{a}{\sqrt{3}} = \frac{0{,}300}{\sqrt{3}}\,\mathrm{Nm} \approx 0.174\,\mathrm{Nm}$$

- The **"acceptance range"** for deviations between the reference value and the indication of the test stations during calibration is $\%\Delta = 0.4\%$ of the full scale value $M_{MAX} = 500$ Nm of the measuring range. Therefore deviations within the limits

  $$a_+ = +\Delta M_\Delta = \frac{\%\Delta}{100\%} \cdot M_{MAX} \qquad \text{and} \qquad a_- = -a_+ = -\Delta M_\Delta = -\frac{\%\Delta}{100\%} \cdot M_{MAX}$$

  must be taken into account, i.e. a maximum of

  $$a = \frac{a_+ - a_-}{2} = \Delta M_\Delta = \frac{\%\Delta}{100\%} \cdot M_{MAX} = \frac{0.4\%}{100\%} \cdot 500\,\mathrm{Nm} = 2.0\,\mathrm{Nm}$$

  The values within the limits of ±2.0 Nm are assumed to be distributed according to a triangular distribution which, unlike the normal distribution, has fixed limits. This assumption is based on the graphical analysis of the measurement errors occurring in practice which were observed during various calibrations of different test stations of identical construction. The corresponding standard uncertainty is calculated according to

  $$u_\Delta = \frac{a}{\sqrt{6}} = \frac{2.0}{\sqrt{6}}\,\mathrm{Nm} \approx 0.817\,\mathrm{Nm}$$

Standard uncertainty of the output quantity

$$\begin{aligned} u_C &= \sqrt{u_R^2 + u_L^2 + u_m^2 + u_\vartheta^2 + u_\Delta^2} \\ &\approx \sqrt{0.015^2 + 0.019^2 + 0.003^2 + 0.174^2 + 0.817^2}\,\mathrm{Nm} \\ &\approx \sqrt{0.000225 + 0.000361 + 0.000009 + 0.030276 + 0.667489}\,\mathrm{Nm} \approx \sqrt{0.698360}\,\mathrm{Nm} \approx 0.836\,\mathrm{Nm} \end{aligned}$$

Expanded measurement uncertainty

The expanded measurement uncertainty is calculated using $k_p = 2$:

$$U = k_p \cdot u_C = 2 \cdot 0.836\,\mathrm{Nm} = 1.672\,\mathrm{Nm} \approx 1.7\,\mathrm{Nm}$$

Complete measurement result

$$M \pm U = M \pm 1.7\,\mathrm{Nm}$$

U applies to measurements close to the reference point $M_0 = 100$ Nm. M denotes the torque value actually indicated by the measuring system.

Information about input quantities							Standard uncertainties of input quantities				Contributions to the measurement uncertainty of the measurand				
Seq. No.	Description	Variable (symbol)	Measuring unit	Value of the variable	Value of the uncertainty data	Comments (z.B. references, explanatory notes, links to documents)	Evaluation type	Type A: Number of measured values; Type B: kp (≥1), confidence level (%), distribution	Numerical factor for calculating the standard uncertainty	Standard uncertainty	Sensitivity coefficient	Contribution to uncertainty	Contribution to uncertainty (squared)	Percentage contribution to MU budget $\frac{(c_i \cdot u(x_i))^2}{\sum_{i=1}^{n}(c_i \cdot u(x_i))^2}$	Rank (according to Pareto)
i				x_i	Δx_i		A B	m_i k_p, %, name	1 or $\sqrt{m_i}$ k_p	$u(x_i) = \Delta x_i / k_p$	c_i	$c_i * u(x_i)$	$(c_i * u(x_i))^2$	[%]	
	Torque (nominal value, reference)	M_0	Nm	100,0											
	Ambient temperature (nominal value, reference)	ϑ_0	°C	20,0											
1	Resolution of torque indication (digit increment)	ΔM_R	Nm	0,05	0,025	Indication uncertain within one "digit increment" (half resolution)	B	Rectangular distribution	1,732	0,015	1	0,015	0,000225	0,032%	4
	Lever arm length (nominal value)	L_0	mm	1.000,0		Manufacturer's specification									
	Maximum deviation of the lever arm length from its nominal value	ΔL	mm	0,32		Value based on manufacturer's specification									
2	Torque error resulting from lever arm length error	ΔM_L	Nm	0,0	0,032	See text; a = $(\Delta L/L_0)*M_0$	B	Rectangular distribution	1,732	0,019	1	0,019	0,000361	0,052%	3
	Maximum deviation of the reference masses from their nominal values	$\%\Delta_W$	%	0,005		Manufacturer's specification									
3	Torque error resulting from reference mass errors	ΔM_W	Nm	0,0	0,005	See text; a = $(\%\Delta_W/100\%)*M_0$	B	Rectangular distribution	1,732	0,003	1	0,003	0,000009	0,001%	5
	Maximum temperature deviation during measurements	$\Delta\vartheta$	K	6,0		Estimation									
	Maximum torque error due to temperature error	$\%\Delta_\vartheta$	%/K	0,1		Manufacturer's specification; reference value M_0									
4	Torque error due to ambient temperature error	ΔM_ϑ	Nm	0,0	0,300	See text; a = $\Delta\vartheta*(\%\Delta_\vartheta/100\%)*M_0$	B	Rectangular distribution	1,732	0,174	1	0,174	0,030276	4,335%	2
	Accepted difference between reference value and indicated value	$\%\Delta$	%	0,4		Setting; reference value M_{MAX}									
	Torque: Full scale value	M_{MAX}	Nm	500,0											
5	Maximum torque error being accepted (acceptance range)	ΔM_Δ	Nm	0,0	2,000	See text; a = $(\%\Delta/100\%)*M_{MAX}$	B	Triangular distribution	2,449	0,817	1	0,817	0,667489	95,580%	1
Model equation:							**Total result:**					$u_c^2 =$	0,698360	100,000%	
$M = M_0 + \delta M_R + \delta M_L + \delta M_m + \delta M_\vartheta + \delta M_\Delta$												$u_c =$	0,836		
												$k_p =$	2,000		
Expected values: $\delta M = 0$			Deviations: $-\Delta M \le \delta M \le \Delta M$									**U =**	**1,672**		

Table 10: Uncertainty budget for the "torque" example

J.5 Optical measurement using a measuring microscope

Description of the measurement

The width of a weld seam is measured manually using microsections and a measuring microscope (10x lens) with an image processing system. Before the measurement, the weld seam of the steel part is severed in its center and a microsection is made. The width of the weld seam is specified as (1.6 ± 0.5) mm, T = 1.0 mm.

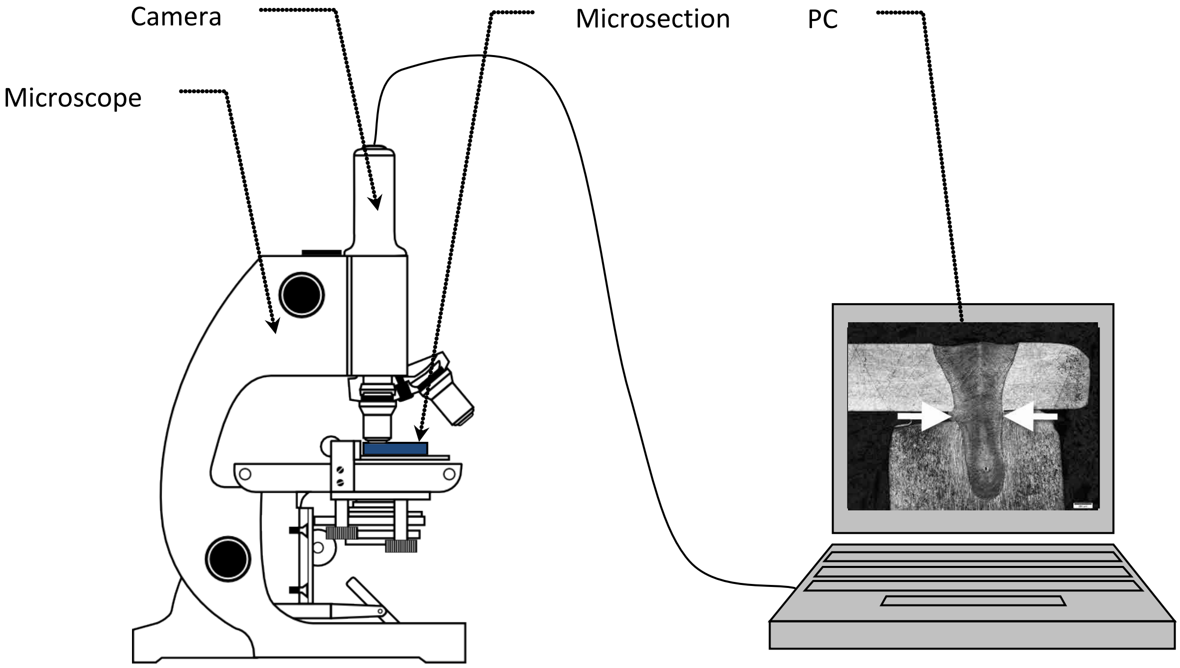

Figure 19: Measurement setup for the optical measurement of microsections

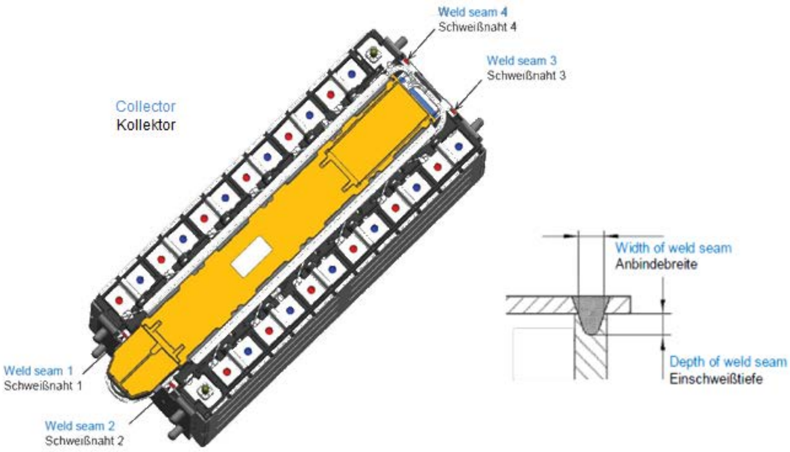

Figure 20: Product part and measuring task (measuring the seam width using a microsection)

The task is to determine the measurement uncertainty according to [ISO 22514-7] and to evaluate the suitability of the measuring system and the measuring process accordingly (cf. chapter 5).

NOTE: Input quantities and model equations are implicitely standardized in case of the approach according to [ISO 22514-7]. The standard does not require any separate specification. Instead, it is sufficient to specify the standard uncertainties of the input quantities according to chapter 5, Table 3 und Table 4, and to calculate the combined output quantities according to the equations (5.1) and (5.2) which correspond to an additive model. Thus, the following sections "input quantities", "model", "measurement results" and "correction" are not mandatory and often omitted in practice. This applies as well to tabular uncertainty budgets.

Input quantities

- **Calibration uncertainty of the calibration plate** (measurement standard) $U_{CAL} = 0.15\ \mu m$
  Data source: DAkkS calibration certificate $k_p = 2$
- **Resolution of the measuring system** $RE = 1.382\ \mu m$
  Data source: Output of the image processing system (software)
- **Repeatability at the standard** $s = 0.919\ \mu m$
  Data source: Standard deviation according to booklet 10, type-1 study
- **Systematic measurement error of the measuring system** $BI = 0.0176\ \mu m$
  Data source: Measurement error according to booklet 10, type-1 study
- **Repeatability of measurement results of the measuring object** $EV = 6.529\ \mu m$
  Data source: EV according to booklet 10, type-2 study
- **Operator impact on measurement results of the measuring object** $AV = 7.298\ \mu m$
  Data source: AV according to booklet 10, type-2 study
- **Interaction between operator and measuring object** $IA = 8.604\ \mu m$
  Data source: IA according to booklet 10, type-2 study

Model (according to chap. 5.2)

Measuring system:

$$y_{MS} = y' + \delta x_{CAL} + \delta x_{EV(MS)} + \delta x_{BI} \tag{J.2}$$

Measuring process:

$$y_{MP} = y_{MS} + \left(\delta x_{EV(MP)} - \delta x_{EV(MS)}\right) + \delta x_{AV} + \delta x_{IA} \tag{J.3}$$

with

y'	Indication for the measurement results y_{MS} of the measuring system or y_{MP} of the measuring process,
δx_{CAL}	Deviation due to the limited precision of calibration,
$\delta x_{EV(MS)}$	Deviation due to the limited repeatability of the measuring system,
δx_{BI}	Systematic measurement error,
$\delta x_{EV(MP)}$	Deviation due to the limited repeatability of the measuring process,
δx_{AV}	Deviation due to operator influence,
δx_{IA}	Deviation due to interactions between input quantities.

Deviations caused by inhomogeneities of the measuring object (δx_{OBJ}) during the measurement (due to setting the positions of measuring points in the measuring microscope based on the operator's visual assessment) are included in the operator influence (δx_{AV}) and the interaction (δx_{IA}) between the operator and the measuring object. Further potential deviations according to [ISO 22514-7] and chap. 5.2, i.e. deviations from linearity (δx_{LIN}), deviations due to instability over time (δx_{STAB}) and temperature influences (δx_{ϑ}), deviations between different measuring systems (δx_{GV}) and deviations due to any other influences ($\delta x_{REST(MS)}, \delta x_{REST(MP)}$) are evaluated as being insignificant or irrelevant. Thus they are not taken into account.

Measurement results

Use of measurement data and evaluation results from type-1 and type-2 studies according to [Booklet 10].

Correction

None

J.5.1 Uncertainties of the measurement system

Standard uncertainties of the measuring system input quantities

- Calibration uncertainty u_{CAL} of the calibration plate from the DAkkS calibration certificate:

$$u_{CAL} = \frac{U_{CAL}}{k_p} = \frac{0.15\,\mu m}{2} = 0.075\,\mu m$$

- Resolution of the measuring system (set by the selected lens, the basic magnification of the camera adapter and the camera, determined and output by the image processing software):

$$u_{RE} = \frac{1}{\sqrt{3}} \frac{RE}{2} = \frac{1}{\sqrt{3}} \cdot \frac{1.382\,\mu m}{2} = 0.399\,\mu m$$

- Repeatability when using a standard (standard deviation s from type-1 study):

$$u_{EVR} = s = 0.919\,\mu m$$

- Determining the measuring system dispersion $u_{EV(MS)}$ from u_{RE} and u_{EVR}:

$$u_{EV(MS)} = MAX(u_{RE}, u_{EVR}) = 0.919\,\mu m$$

- Systematic measurement error (bias from type-1 study):

$$u_{BI} = \frac{\left|\bar{x} - x_m\right|}{\sqrt{3}} = \frac{0.0176\,\mu m}{\sqrt{3}} = 0.0102\,\mu m$$

Other uncertainties are evaluated as insignificant.

Combined standard uncertainty of the measuring system

$$u_{MS} = \sqrt{u_{CAL}^2 + u_{EV(MS)}^2 + u_{BI}^2} = \sqrt{(0.075\,\mu m)^2 + (0.919\,\mu m)^2 + (0.0102\,\mu m)^2} = 0.922\,\mu m$$

Expanded measurement uncertainty of the measuring system

$$U_{MS} = k_p \cdot u_{MS} = 2 \cdot 0.922\,\mu m = 1.844\,\mu m$$

Evaluation of the measuring system

$$Q_{MS} = \frac{2 \cdot U_{MS}}{T} \cdot 100\% = \frac{2 \cdot 1.844\,\mu m}{1000\,\mu m} \cdot 100\% = 0.37\% \le 15\%$$

Result: The measuring system is suitable ($Q_{MS} \le 15\%$).

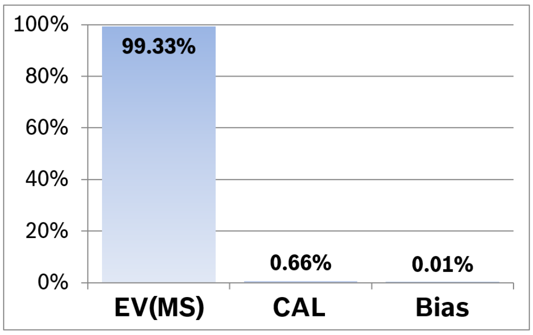

Figure 21: Pareto chart of uncertainty contributions u_i^2 to the uncertainty of the measuring system

Information about input quantities							Standard uncertainties of input quantities				Contributions to the measurement uncertainty of the measurand				
Seq. No.	Description	Variable (symbol)	Measuring unit	Value of the variable	Value of the uncertainty data	Comments (z.B. references, explanatory notes, links to documents)	Evaluation type	Type A: Number of measured values; Type B: kp (≥1), confidence level (%), distribution	Numerical factor for calculating the standard uncertainty	Standard uncertainty	Sensitivity coefficient	Contribution to uncertainty	Contribution to uncertainty (squared)	Percentage contribution to MU budget $\frac{(c_i \cdot u(x_i))^2}{\sum_{i=1}^{n}(c_i \cdot u(x_i))^2}$	Rank (according to Pareto)
i				x_i	Δx_i		A B	m_i k_p, %, name	1 or $\sqrt{m_i}$ k_p	$u(x_i) = \Delta x_i / k_p$	c_i	$c_i * u(x_i)$	$(c_i * u(x_i))^2$	[%]	
1	Uncertainty of calibration	δx_{CAL}	µm	0	0,15	U = 0,15µm and k_p = 2 according to calibration certificate no. Nr. 12345	B	2,000	2,000	0,0750	1	0,0750	0,00562500	0,66%	2
	Resolution of the measuring system	x_{RE} = RE	µm	1,382	0,691	$\Delta x_{RE} = x_{RE} / 2$ = RE / 2; RE according to output of image processing system	B	Rectangular distribution	1,732	0,3990					
	Repeatability of the measurement results when using a standard	δx_{EVR} = s	µm	0	0,919	Standard deviation according to booklet 10, type-1 study	A		1,000	0,9190					
2	Dispersion of the measuring system when using a standard	$\delta x_{EV(MS)}$	µm	0	*is calculated*	Maximum of the **standard uncertainties** determined from RE and EVR				0,9190	1	0,9190	0,84456100	99,33%	1
3	Systematic measurement error	δx_{BI} = BI	µm	0	0,0176	Measurement error according to booklet 10, type-1 study	B	Rectangular distribution	1,732	0,0102	1	0,0102	0,00010404	0,01%	3
4															
5															
6															
7															
8															
9															
Model equation: Messsystem:							Total result:					u_{MS}^2 =	0,850	100,00%	
$y_{MS} = y_0 + \delta x_{CAL} + \delta x_{EV(MS)} + \delta x_{BI}$												u_{MS} =	0,922		
Expected values: $\delta x = 0$			Deviations: $\Delta x \le \delta x \le \Delta x$									k_p =	2,000		
												U_{MS} =	**1,844**		

Table 11: “Microscope” example according to [ISO 22514-7]; uncertainty budget “measuring system”

J.5.2 Uncertainties of the measuring process

Standard uncertainties of the measuring process input quantities

- Standard uncertainty of the measuring system (u_{MS} from chap. J.5.1):
  $u_{MS} = 0.922\ \mu m$
- Repeatability on the measuring object (EV from type-2 study):
  $u_{EVO} = EV = 6{,}529\ \mu m$
- Determining $u_{EV(MP)}$ from u_{RE}, u_{EVR} and u_{EVO}:
  $u_{EV(MP)} = MAX(u_{RE}, u_{EVR}, u_{EVO}) = 6.529\ \mu m$
- Reproducibility, operator influence (AV from type-2 study):
  $u_{AV} = AV = 7.298\ \mu m$
- Interaction (IA from type-2 study):
  $u_{IA} = IA = 8.604\ \mu m$

Other uncertainties are evaluated as insignificant.

Combined standard uncertainty of the measuring process

$$u_{MP} = \sqrt{u_{MS}^2 + \left(u_{EV(MP)}^2 - u_{EV(MS)}^2\right) + u_{AV}^2 + u_{IA}^2}$$
$$= \sqrt{(0.922\ \mu m)^2 + \left((6.529\ \mu m)^2 - (0.919\ \mu m)^2\right) + (7.298\ \mu m)^2 + (8.604\ \mu m)^2} = 13.035\ \mu m$$

Expanded measurement uncertainty of the measuring process

$$U_{MP} = k_p \cdot u_{MP} = 2 \cdot 13.035\,\mu m = 26.070\,\mu m$$

Evaluation of the measuring process

$$Q_{MP} = \frac{2 \cdot U_{MP}}{T} \cdot 100\% = \frac{2 \cdot 26.070\,\mu m}{1000\,\mu m} \cdot 100\% = 5.21\% \leq 30\%$$

Result: The measuring process is suitable ($Q_{MP} \leq 30\%$).

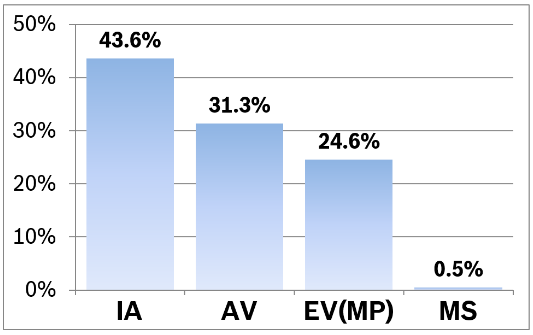

Figure 22: Pareto chart of uncertainty contributions u_i^2 to the uncertainty of the measuring process

NOTE: $u^2_{EV(MP)}$ cleaned up, i.e. without $u^2_{EV(MS)}$ which is already included in u^2_{MS}.

Information about input quantities							Standard uncertainties of input quantities				Contributions to the measurement uncertainty of the measurand				
Seq. No.	Description	Variable (symbol)	Measuring unit	Value of the variable	Value of the uncertainty data	Comments (z.B. references, explanatory notes, links to documents)	Evaluation type	Type A: Number of measured values; Type B: kp (≥1), confidence level (%), distribution	Numerical factor for calculating the standard uncertainty	Standard uncertainty	Sensitivity coefficient	Contribution to uncertainty	Contribution to uncertainty (squared)	Percentage contribution to MU budget $\frac{(c_i \cdot u(x_i))^2}{\sum_{i=1}^{n}(c_i \cdot u(x_i))^2}$	Rank (according to Pareto)
i				x_i	Δx_i		A B	m_i k_p, %, name	1 or $\frac{\sqrt{m_i}}{k_p}$	$u(x_i) = \Delta x_i / k_p$	c_i	$c_i * u(x_i)$	$(c_i * u(x_i))^2$	[%]	
1	Measurement results of the measuring system	y_{MS}	µm		1,844	MU budget of the measuring system, expanded MU U_{MS}	B	2,000	2,000	0,9220	1	0,9220	0,850084	0,5%	4
	Measuring system dispersion when using a standard	$\delta x_{EV(MS)}$	µm	0	0,919	MU budget of the measuring system, fraction of dispersion contained in in U_{MS}	B		1,000	0,9190					
	Repeatability of the measurement results when using the measuring object	$\delta x_{EVO} = EV$	µm	0	6,529	EV according to booklet 10, type-2 study	A		1,000	6,5290					
	Measuring system dispersion when using measuring objects	$\delta x_{EV(MP)}$	µm	0	*is calculated*	Maximum of the **standard uncertainties** determined from EV(MS) and EVO				6,5290					
3	Increase of measuring system dispersion resulting from the measuring object	$\delta x_{EV(MP)} - \delta x_{EV(MS)}$	µm	0	*is calculated*	$u_{EV(MP)}$ without the fraction contained in u_{MS}: $\sqrt{u_{EV(MP)}^2 - u_{EV(MS)}^2}$				6,4640	1	6,4640	41,783296	24,6%	3
4	Operator influence on measurement results of the measuring object	$\delta x_{AV} = AV$	µm	0	7,298	AV according to booklet 10, type-2 study	A		1,000	7,2980	1	7,2980	53,260804	31,3%	2
5	Interaction between operator and measuring object	$\delta x_{IA} = IA$	µm	0	8,604	IA according to booklet 10, type-2 study	A		1,000	8,6040	1	8,6040	74,028816	43,6%	1
6															
9															
Model equation: Messprozess:							**Total result:**					$\mathbf{u_{MP}^2} =$	169,923	100,0%	
$y_{MP} = y_{MS} + (\delta x_{EV(MP)} - \delta x_{EV(MS)}) + \delta x_{AV} + \delta x_{IA}$												$\mathbf{u_{MP}} =$	13,035		
												$\mathbf{k_p} =$	2,000		
Expected values: $\delta x = 0$			Deviations: $\Delta x \le \delta x \le \Delta x$									$\mathbf{U_{MP}} =$	**26,070**		

Table 12: “Microscope” example according to [ISO 22514-7]; uncertainty budget “measuring process”

J.6 In-process tactile diameter measurement

Description of the measurement

During shaft production the process step "grinding" is monitored by tactile sampling inspections of the shaft diameter. The operator places the shaft to be tested in a horizontal position between tip-shaped brackets (briefly "tips"). After that, the shaft surface is scanned fully automatically by the measuring system and the shaft diameter is determined from the measured data.

The capability of the measuring process is proven by means of type-1 and type-3 studies [Booklet 10]. For continuous monitoring of the measuring process stability, a calibrated series part (a so-called "stability part") is measured in exactly the same way as series parts and a measurement stability chart is maintained according to a type-5 study [Booklet 10]. The calibration certificate of the "stability part" provides the uncertainty of the calibration of this standard.

The data from the calibration certificates and the procedures according to [Booklet 10] are used to determine the uncertainty of the results of the measuring process which is updated ongoing during production.

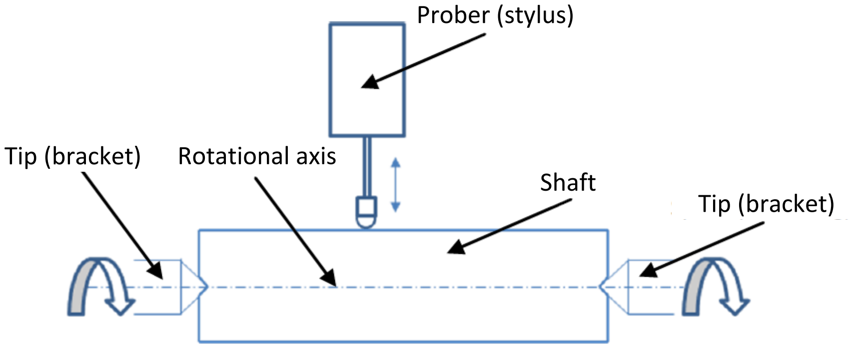

Figure 23: Principle of tactile measurement of a shaft diameter

Input quantities

- **Reference value of the standard** (calibration certificate) $x_{CAL} = 36457.1\,\mu m$
- **Calibration uncertainty of the standard** (calibration certificate) $U_{CAL} = 1.7\,\mu m\,;\ k_p = 2$
- **Resolution of indication** (digit increment) $\delta x_{RE} \leq 0.5$ Digit

  NOTE 1: The uncertainty of a measurement result basically cannot be less than the resolution of the measuring system. In the present case the resolution is determined by the indication of the measuring system. Therefore, it is already included in the deviations of the measured values from the respective conventional value and must not be considered separately once again.

- **Mean value of the (uncorrected) measured values** $\bar{x} = 36457.476\,\mu m$
  Data source: Stability chart according to booklet 10, type-5 study
- **Standard deviation of the measured values** $s_x = 10.125\,\mu m$
  Data source: Stability chart according to booklet 10, type-5 study

  NOTE 2: Dispersion is caused by all influences affecting in total the measuring process including their interactions, finite repeatability of the measuring system and measuring process, operator influence, finite long-term stability, temperature fluctuations, as well as other factors that are not caused by the measured parts such as vibrations in the manufacturing environment. These effects are taken into account to the extent they are contained in the last 25 values of the stability charts.

- **Deviation due to impact of parts**
  Data source: Results of type-1 and type-3 studies according to booklet 10;
  Determination from:
  - Standard deviation from type-1 study: $s = 0.139\,\mu m$
  - Measuring system dispersion from type-3 study: $EV = 0.131\,\mu m$

  NOTE 3: Deviations are caused by the different nature of the standard ("stability part") and the serial parts.

Model

$$y = y' + \delta x_{CAL} + \delta x_{BI} + \delta x_{PRO} + \delta x_{PAR}$$

with

y	(Current) indication for the diameter,
y'	Uncorrected average indication (mean value of the stability chart),
δx_{CAL}	Deviation due to the limited precision of the calibration of the standard,
δx_{BI}	Deviation due to the uncorrected systematic measurement error,
δx_{PRO}	Deviation due to the measurement procedure,
δx_{PAR}	Deviation due to the difference between the standard and series parts.

$-\Delta x \le \delta x \le \Delta x$ applies to all above-mentioned deviations. Here, δx describes the instantaneous value of the fluctuating deviation (expected value $\delta x = 0$), Δx the associated maximum deviation.

Measurement results

Use of measured data and evaluation results of type-1, type-3 and type-5 studies according to [Booklet 10].

Correction

None.

NOTE 4: Systematic measurement errors are considered to be a standard uncertainty u_{BI} in the uncertainty budget (cf. chapter 6.1.2 and appendix F.3).

Standard uncertainties of the input quantities

- **Uncertainty u_{CAL} of the calibration of the standard used**
  The calibration certificate of the standard provides the expanded measurement uncertainty U_{CAL} = 1.70 µm and the coverage factor k_p = 2. The corresponding standard uncertainty is calculated as

  $$u_{CAL} = \frac{U_{CAL}}{k_p} = \frac{1.70}{2}\,\mu m = 0.85\,\mu m$$

- **Uncertainty u_{RE} due to the limited resolution of the indication**
  As already explained, the corresponding deviations are included in the measured values and thereby taken into account via the uncertainty u_{PRO} of the measurement procedure. So, there is no need to consider a separate standard uncertainty u_{RE}.

- **Uncertainty u_{BI} due to uncorrected systematic errors (“bias”)**
  The systematic error is calculated as the difference of the mean value $\bar{x}$ calculated from 25 measured values recorded in the stability charts of the recent weeks and the reference value x_{CAL} of the standard:
  $\Delta x_{BI} = \left| \bar{x} - x_{CAL} \right| = 36457.476\ \mu m - 36457.100\ \mu m = 0.376\ \mu m$
  Systematic errors that are not compensated by correction must be included in the measurement uncertainty as a standard uncertainty (see appendix F.3):
  $u_{BI} = \Delta x_{BI} = 0.376\ \mu m$

- **Uncertainty u_{PRO} due to the measurement procedure**
  The standard uncertainty of the measurement procedure is calculated as standard deviation s_X of the last 25 values x documented in the stability charts:
  $u_{PRO} = s_x = 10.125\ \mu m$

  NOTE 5: The measurement uncertainty U to be determined is intended to allow a statement about the respective individual measured value. Accordingly, for u_{PRO}, the standard deviation s of the individual measured values from their mean value $\bar{x}$ must be used (rather than the standard deviation of the mean value that is smaller by the factor $1/\sqrt{25}$).

- **Uncertainty u_{PAR} due to measured parts**
  Deviations caused by the different nature of the standard (i.e. the “stability part”) and the series parts must be considered to be significant and included in the measurement uncertainty only if the following condition is fulfilled (cf. chapter 6.1.4):
  $EV^2 > 2 \cdot s^2$
  With EV = 0.131 µm from a type-3 study and s = 0.139 µm from a type-1 study:
  $EV^2 = (0.131\ \mu m)^2 = 0.017161\ \mu m^2 < 2 \cdot s^2 = 2 \cdot (0.139\ \mu m)^2 = 2 \cdot 0.019321\ \mu m^2 = 0.038642\ \mu m^2$
  Therefore, the significance condition is not met so that the uncertainty u_{PAR} is negligible:
  $u_{PAR} = 0\ \mu m$

  NOTE 6: Reports of measuring process analyses often specify %EV instead of EV. Then, %EV must be multiplied by the reference value in order to calculate EV. The reference value is often the tolerance of the characteristic, but may be also another quantity. This must be clarified if necessary.

<u>Standard uncertainty of the output quantity</u>

$$
\begin{aligned}
u_C &= \sqrt{u_{CAL}^2 + u_{BI}^2 + u_{PRO}^2 + u_{PAR}^2} \\
&\approx \sqrt{0.850^2 + 0.376^2 + 10.125^2 + 0^2}\ \mu m \\
&\approx \sqrt{0.722500 + 0.141376 + 102.515625 + 0}\ \mu m \approx \sqrt{103.379471}\ \mu m \approx 10.168\ \mu m
\end{aligned}
$$

<u>Expanded measurement uncertainty</u>

The expanded measurement uncertainty is calculated using $k_p = 2$:

$$U = k_p \cdot u_C = 2 \cdot 10.168\ \mu m = 20.336\ \mu m$$

<u>Complete measurement result</u>

$$y = y' \pm U = y' \pm 20.336\ \mu m$$

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Information about input quantities							Standard uncertainties of input quantities				Contributions to the measurement uncertainty of the measurand				
Seq. No.	Description	Variable (symbol)	Measuring unit	Value of the variable	Value of the uncertainty data	Comments (z.B. references, explanatory notes, links to documents)	Evaluation type	Type A: Number of measured values; Type B: kp (≥1), confidence level (%), distribution	Numerical factor for calculating the standard uncertainty	Standard uncertainty	Sensitivity coefficient	Contribution to uncertainty	Contribution to uncertainty (squared)	Percentage contribution to MU budget $\frac{(c_i \cdot u(x_i))^2}{\sum_{i=1}^{n}(c_i \cdot u(x_i))^2}$	Rank (according to Pareto)
i				x_i	Δx_i		A B	m_i k_p, %, name	1 or $\sqrt{m_i}$ k_p	$u(x_i) = \Delta x_i / k_p$	c_i	$c_i * u(x_i)$	$(c_i * u(x_i))^2$	[%]	
1	Reference value of the standard	x_{CAL}	µm	36457,1	1,7	Calibration certificate no. 12345	B	Normal distribution	2,000	0,850	1,000	0,850	0,722500	0,70%	2
2	Resolution of indication	x_{RE}	Not considered here since contained in experimentally determined "deviation resulting from measurement procedure"												
3	Uncorrected systematic measurement error	x_{BI}	µm		0,376	Stability chart: Deviation mean value ⇔ reference value of the standard	A		1,000	0,376	1,000	0,376	0,141376	0,14%	3
4	Deviation resulting from measurement procedure	x_{PRO}	µm		10,125	Stability chart: Standard deviation of the individual values	A		1,000	10,125	1,000	10,125	102,515625	99,16%	1
5	Deviation resulting from part impact	x_{PAR}	µm		0	Insignificant contribution (see text)	A		1,000	0,000	1,000	0,000	0,000000	0,00%	4
6															
7															
8															
9															
10															
11															
Model equation:							**Total result:**					**u_c^2 =**	103,380	100,000%	
$y = y' + \delta x_{CAL} + \delta x_{BI} + \delta x_{PRO} + \delta x_{PAR}$												**u_c =**	10,168		
												k_p =	2,000		
Expected values: $\delta x = 0$			Deviations: $\Delta x \le \delta x \le \Delta x$									**U =**	**20,336**		

Table 13: Uncertainty budget for the "shaft diameter" example based on stability charts

J.7 Injection quantity indicator (EMI)

The injection quantity indicator (or briefly EMI according to German ***E**inspritz**m**engen**i**ndikator*) measures injected masses (coll. also called *injection quantities*). The calibration uncertainty is to be determined.

Description of the measurement

The mass of test oil injected into the EMI working chamber (e.g. diesel fuel) displaces a piston. An inductive measuring system records the path x traveled by the piston. The injected mass m (output quantity) is calculated from the measured path x, the cross-sectional area A of the piston and the density ρ of the test oil (input quantities). The pressure p and the temperature ϑ inside the EMI chamber must be considered as well. The calculated injection mass m is adjusted by means of a correction value k_f to the indication m_0 of the standard device (scale) which directly measures the actually injected mass. In fact, the measured path x is rescaled into the injected mass m.

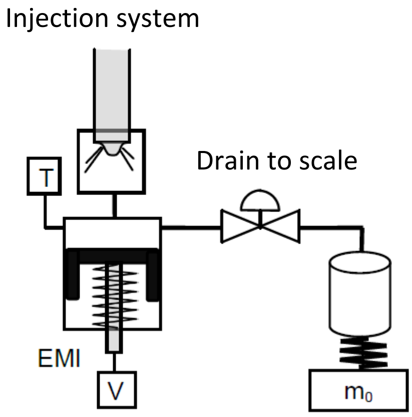

Figure 24: Measuring principle for adjustment and calibration of an injection quantity indicator (EMI)

Because of the limited sensitivity and resolution of the standard device (scale), a sufficiently large mass of the test medium is required for each weighing process. Therefore, the **total mass m** of n = 1000 injection processes is weighed. The balancing between the EMI and the scale is based on the measurement results of the total mass m, rather than the (calculated) mean values for a single injection process.

Basic equation for determining the injection mass m from the injection volume V:

$$m = \rho \cdot V \tag{J.4}$$

with

$\rho(\vartheta, p)$	Volume density of the injected medium at temperature ϑ and pressure p,
$V = x \cdot A$	Chamber volume which is displaced by the injected mass,
x	Piston travel,
$A = \pi \cdot \left(\frac{d + k_f}{2}\right)^2$	Piston area,
d	Piston diameter (data sheet),
k_f	Correction value (result of the adjustment),

so that

$$m = \rho(\vartheta, p) \cdot x \cdot \pi \cdot \left(\frac{d + k_f}{2}\right)^2 . \tag{J.5}$$

The correction value k_f is determined by comparison with a standard device (scale). The indication m of the EMI is adjusted to the indication m_0 of the scale, i.e.

$$m = m_0\,, \tag{J.6}$$

or equation (J.5) substituted for m

$$\rho(\vartheta, p) \cdot x \cdot \pi \cdot \left(\frac{d + k_f}{2}\right)^2 = m_0 \tag{J.7}$$

and solved for k_f yields

$$k_f = 2 \cdot \sqrt{\frac{m_0}{\rho(\vartheta, p) \cdot x \cdot \pi}} - d \tag{J.8}$$

This additive correction value k_f for the piston diameter is the result of the **adjustment**. In relation to the EMI indication, k_f effectively provides a (non-linear) correction of the deviation of the EMI indication from the scale indication, of the test medium density, of the piston travel and of the piston diameter at the time of adjustment. The value determined is added to the EMI configuration data (flash EEPROM). Therefore, k_f represents a parameter which is invariant until the next adjustment and equally impressed to all EMI measurement results for the "injection mass". The uncertainty of this correction must be considered in the uncertainty analysis.

Subsequently, the determined correction value k_f is used to perform another comparison of the EMI measuring instrument with the scale at the calibration point (200 g), i.e. a **calibration** is carried out.

Input quantities

- **Temperature ϑ in the EMI measurement chamber:**
  The temperature ϑ is measured using a calibrated thermocouple. The measurement result is adversely affected by a measurement deviation $\delta\vartheta$ of the thermocouple in the installed state which results from its calibration. $|\delta\vartheta| \le 0.5\ \mathrm{K}$
- **Pressure p in the EMI measurement chamber:**
  Pressure differences within the EMI are disregarded. $|\delta p| \approx 0\ \mathrm{bar}$
- **Volume density $\rho(\vartheta,p)$ of the test medium:**
  The density at the measured EMI chamber temperature ϑ and the atmospheric pressure p is determined by linear interpolation from the densities measured at the reference temperatures ϑ_1 and ϑ_2.
  - Reference temperature #1: $\vartheta_1 = 20\ ^\circ\mathrm{C}$
  - Measured density at reference temperature #1: $\rho_1 = 0.820\ \frac{\mathrm{g}}{\mathrm{cm}^3}$
  - Reference temperature #2: $\vartheta_2 = 80\ ^\circ\mathrm{C}$
  - Measured density at reference temperature #2: $\rho_2 = 0.778\ \frac{\mathrm{g}}{\mathrm{cm}^3}$
  - Uncertainties $\delta\vartheta$ and $\delta\rho$ of the reference points (ϑ_1; ρ_1) and (ϑ_2; ρ_2) as well as deviations of the function $\rho(\vartheta)$ from a straight line are evaluated as negligible.
  - Density variations $\delta\rho$ due to pressure fluctuations δp are considered to be negligible $|\delta\rho(\vartheta, \delta p)| \approx 0\ \frac{\mathrm{g}}{\mathrm{cm}^3}$

- **(Uncorrected) volume V' of the EMI measurement chamber:**
  The piston travel is measured using an LVDT (Linear Variable Differential Transformer). Plotting the determined values versus the reference values of the travel measuring system results in an S-shaped curve. The S-shape is corrected using a correction table provided by the EMI manufacturer so that a linearized characteristic curve of the LVDT is obtained. The deviation resulting from this linearization for each injection process is specified in the data sheet of the EMI manufacturer as a deviation $\delta V'$ from the (uncorrected) nominal volume V' of the EMI chamber. $\left|\delta V'\right| \le 0.1\,\text{mm}^3$

- **(Uncorrected) indication m' of the EMI:**
  It is assumed that the measurand is adversely affected by a measurement error which is caused particularly by the dispersion of the injected mass rather than the linearization of the LVDT characteristic curve. This deviation is estimated based on the standard deviation of n_M repeated measurements (measured values x_i see Table 14).
  $n_M = 5$
  $$s = \sqrt{\frac{1}{n_M - 1}\sum_{i=1}^{n_M}(x_i - \bar{x})^2}$$

- **Diameter d of the EMI piston:**
  The diameter is assumed to be constant at d = 16.97 mm (mean value known from production). Deviations δd due to individual dispersion are contained in the correction value k_f.
  - Piston diameter $d = 1.697$ cm
  - Individual dispersion $\left|\delta d\right| \approx 0$ cm

- **Measurement uncertainty of the scale:**
  The measurement uncertainty of the standard device (scale) is specified by the calibration laboratory.
  - Reference value $m_0 = 200$ g
  - Expanded measurement uncertainty ($k_p = 2$) $U_0 = 0.184$ g

- **Number of injections n per measurement result:**
  It is always the total mass of n injection processes which is weighed. It must be ensured for this purpose that always exactly n injections are evaluated.
  - Number of injections per weighing operation $n = 1000$
  - Deviations from the nominal number of injections $\left|\delta n\right| = 0$

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J.7.1 Adjustment and uncertainty of the EMI measuring instrument

Model equation

The model equation is given by Eq. (J.5). In this form the equation includes the piston travel x and the correction factor k_f as input quantities. However, information about uncertainties is not immediately available for these quantities. This fact usually complicates the calculations significantly. Therefore, it is advantageous to transform the model equation algebraically and to represent it as far as possible using quantities with directly available uncertainty data.

First, Eq. (J.8) for k_f is transformed. Expanding the term under the root operator with $(d/2)^2$ and defining the uncorrected EMI indication m' and the EMI chamber volume V' according to

$$m' = \rho(T,p) \cdot V' = \rho(\vartheta,p) \cdot x \cdot \pi \cdot \left(\frac{d}{2}\right)^2 \tag{J.9}$$

results in

$$k_f = 2 \cdot \sqrt{\frac{m_0}{\rho(\vartheta,p) \cdot x \cdot \pi}} - d = \left(\sqrt{\frac{m_0}{m'}} - 1\right) \cdot d \tag{J.10}$$

Solving Eq. (J.10) for $(d + k_f)/2$ and substituting it in the model equation Eq. (J.5) yields

$$m = \rho \cdot x \cdot \pi \cdot \left(\frac{d + k_f}{2}\right)^2 = \rho \cdot x \cdot \pi \cdot \left(\sqrt{\frac{m_0}{m'}} \cdot \frac{d}{2}\right)^2 = \rho \cdot x \cdot \pi \cdot \left(\frac{d}{2}\right)^2 \cdot \frac{m_0}{m'} = \rho \cdot V' \cdot \frac{m_0}{m'} \tag{J.11}$$

This equation represents the corrected EMI indication m exclusively dependent on input quantities providing uncorrected measured values which are directly readable as well as uncertainty data which are independent of each other.

Measurement results

Measurement no.		1	2	3	4	5	Mean value	Standard deviation
Scale indication	m_0 / g	200.35	200.40	200.42	200.44	200.45	200.412	0.039623
EMI indication (uncorrected)	m' / g	200.24	200.24	200.28	200.32	200.31	200.278	0.037683
EMI chamber temperature	ϑ / °C	67.30	67.45	67.40	67.33	67.40	67.376	0.060249

Table 14: Scale and EMI indications for injected masses and measured EMI chamber temperature
(Mass of 1000 individual injection operations added up in each case)

Correction (of the adjustment)

With the above input quantities, the linearly interpolated volume density at the mean EMI chamber temperature $\overline{\vartheta} = 67{,}376\ °C$

$$\begin{aligned} \rho(\vartheta,p) &= \frac{\rho(\vartheta_2,p) - \rho(\vartheta_1,p)}{T_2 - T_1} \cdot (\vartheta - \vartheta_1) + \rho(\vartheta_1,p) \\ &= \frac{0.778\ \frac{g}{cm^3} - 0.820\ \frac{g}{cm^3}}{80\ °C - 20\ °C}(67.376\ °C - 20\ °C) + 0.820\ \frac{g}{cm^3} = 0.786837\ \frac{g}{cm^3} \end{aligned} \tag{J.12}$$

and the mean values $\overline{m}_0$ and $\overline{m}'$ of the measurement data, the correction k_f is calculated according to Eq. (J.10):

$$k_f = \left(\sqrt{\frac{\overline{m}_0}{\overline{m}'}} - 1\right) \cdot d = \left(\sqrt{\frac{200.412\ g}{200.278\ g}} - 1\right) \cdot 1.697\ cm = 0.000568\ cm \tag{J.13}$$

Standard uncertainties of the input quantities

- **Uncertainty due to the temperature ϑ in the measurement chamber**
  Because of a lack of more precise knowledge, the standard uncertainty is determined from the calibration uncertainty of the thermocouple assuming a rectangular distribution:
  $$u_\vartheta = \frac{|\delta\vartheta|}{\sqrt{3}} = \frac{0.5}{\sqrt{3}}\,\mathrm{K} = 0.288676\,\mathrm{K}$$
  The temperature affects the volume density of the test medium. The associated sensitivity coefficient is calculated according to
  $$c_\vartheta = \frac{\partial m}{\partial\rho}\frac{\partial\rho}{\partial\vartheta} = \frac{\partial}{\partial\rho}\left(\rho\cdot V'\cdot\frac{m_0}{m'}\right)\cdot\frac{\partial\rho}{\partial\vartheta} = V'\cdot\frac{m_0}{m'}\cdot\frac{\delta\rho}{\delta\vartheta} = \frac{m_0}{\rho(\vartheta,p)}\cdot\frac{\rho(\vartheta_2,p)-\rho(\vartheta_1,p)}{\vartheta_2-\vartheta_1}$$
  $$= \frac{200.412\,\mathrm{g}}{0.786837\,\frac{\mathrm{g}}{\mathrm{cm}^3}}\cdot\frac{0.778\,\frac{\mathrm{g}}{\mathrm{cm}^3}-0.820\,\frac{\mathrm{g}}{\mathrm{cm}^3}}{80\,^\circ\mathrm{C}-20\,^\circ\mathrm{C}} = -0.178294\,\frac{\mathrm{g}}{\mathrm{K}}$$
  Here, the relationship $m' = \rho\cdot V'$ is used. For $\rho(\vartheta,p)$ the interpolated value is used which is calculated according to Eq. (J.12) for $\overline{\vartheta} = 67{,}376\,^\circ\mathrm{C}$. For m_0 the mean value $\overline{m}_0$ of the scale indications is used. The term $\partial\rho/\partial\vartheta$ is approximated by the slope of the straight line used for the linear interpolation of the volume density.

- **Uncertainty due to the pressure in the measurement chamber**
  Because of $|\delta p| \approx 0\,\mathrm{bar}$, $u_p = 0\,\mathrm{bar}$ is assumed. Thus, there is no need to calculate the sensitivity coefficient.

- **Uncertainty due to the (uncorrected) volume V‘ of the measurment chamber**
  The standard uncertainty is calculated based on the manufacturer's specifications assuming a normal distribution:
  $$u_{V'} = \frac{|\delta V'|}{2} = \frac{0.1}{2}\,\mathrm{cm}^3 = 0.05\,\mathrm{cm}^3$$
  The associated sensitivity coefficient is calculated according to
  $$c_{V'} = \frac{\partial m}{\partial V'} = \frac{\partial}{\partial V'}\left(\rho\cdot V'\cdot\frac{m_0}{m'}\right) = \rho(\vartheta,p)\cdot\frac{m_0}{m'} = 0.786837\,\frac{\mathrm{g}}{\mathrm{cm}^3}\cdot\frac{200.412\,\mathrm{g}}{200.278\,\mathrm{g}} = 0.787363\,\frac{\mathrm{g}}{\mathrm{cm}^3}$$
  For $\rho(\vartheta,p)$ the interpolated value is used which is calculated according to Eq. (J.12) for $\overline{\vartheta} = 67.376\,^\circ\mathrm{C}$. For m_0 the mean value $\overline{m}_0$ of the scale indications is used, for m' the mean value $\overline{m}'$ of the uncorrected EMI indications.

- **Uncertainty due to limited repeatability of the (uncorrected) EMI indications m‘**
  The uncertainty is determined using the standard deviation of the EMI indications:
  $$s_{m'} = \sqrt{\frac{1}{n_M-1}\sum_{i=1}^{n_M}\left(m'_i-\overline{m}'\right)^2}$$
  The measured values given in Table 14 and $n_M = 5$ result in
  $s_{m'} = 0.037683\,\mathrm{g}$.
  The corresponding standard deviation of the mean value is used as standard uncertainty:
  $$u_{m'} = \frac{s_{m'}}{\sqrt{n_m}} = \frac{0.037683\,\mathrm{g}}{\sqrt{5}} = 0.016853\,\mathrm{g}$$
  The following applies to the associated sensitivity coefficient:
  $$c_{m'} = \frac{\partial m}{\partial m'} = \frac{\partial}{\partial m'}\left(\rho\cdot V'\cdot\frac{m_0}{m'}\right) = \rho\cdot V'\cdot\left(-\frac{m_0}{m'^2}\right) = m'\cdot\left(-\frac{m_0}{m'^2}\right) = -\frac{m_0}{m'} = -\frac{200.412\,\mathrm{g}}{200.278\,\mathrm{g}} = -1.000669$$
  Here, the relationship $m' = \rho\cdot V'$ is used. For m_0 the mean value $\overline{m}_0$ of the scale indications is used, for m' the mean value $\overline{m}'$ of the uncorrected EMI indications.

- **Uncertainty due to deviations from the nominal diameter d of the piston**
  Because of $|\delta d| \approx 0\ \text{mm}$, $u_d = 0\ \text{mm}$ is assumed. Thus, there is no need to calculate the sensitivity coefficient.

- **Uncertainty owing to deviations from the nominal number n of injections**
  Because of $|\delta n| \approx 0$, $u_n = 0$ is assumed. Thus, there is no need to calculate the sensitivity coefficient.

- **Uncertainty of the indications m_0 of the standard device (scale)**
  - **Measurement uncertainty of the weighing process**
    The standard uncertainty is calculated from the data available for the expanded measurement uncertainty U_0 and for the coverage factor k_p of the scale:
    $$u_0 = \frac{U_0}{k_p} = \frac{0.184\ \text{g}}{2} = 0.092\ \text{g}$$
    The following applies to the sensitivity coefficient:
    $$c_0 = \frac{\partial m}{\partial m_0} = \frac{\partial}{\partial m_0}\left(\rho \cdot V' \cdot \frac{m_0}{m'}\right) = \rho \cdot V' \cdot \left(\frac{1}{m'}\right) = m' \cdot \left(\frac{1}{m'}\right) = 1$$
  - **Uncertainty due to limited repeatability of the measurement results (dispersion)**
    It is assumed that the dispersion fraction which has to be considered as a property of the scale (inherent dispersion) is taken into account in the calibration uncertainty U_0. It is further assumed that dispersion fractions going beyond this can be attributed to the dispersion of the injection masses in the EMI chamber, so that they are already taken into account in the dispersion of the EMI indications.

Standard uncertainty of the output quantity: Corrected EMI indication for the injection mass m

$$u_m = \sqrt{(c_\vartheta \cdot u_\vartheta)^2 + (c_{V'} \cdot u_{V'})^2 + (c_{m'} \cdot u_{m'})^2 + (c_0 \cdot u_0)^2}$$

$$\approx \sqrt{\left(-0.178294\ \frac{\text{g}}{°\text{C}} \cdot 0.288676\ °\text{C}\right)^2 + \left(0.787363\ \frac{\text{g}}{\text{cm}^3} \cdot 0.050000\ \text{cm}^3\right)^2 + (-1.000669 \cdot 0.016853\ \text{g})^2 + (1.000000 \cdot 0.092000\ \text{g})^2}$$

$$\approx \sqrt{(-0.051470)^2 + 0.039369^2 + 0.016865^2 + 0.092000^2}\ \text{g}$$

$$\approx \sqrt{0.002649160900 + 0.001549918161 + 0.000284428225 + 0.008464000000}\ \text{g}$$

$$\approx \sqrt{0.012948}\ \text{g} \approx 0.113789\ \text{g}$$

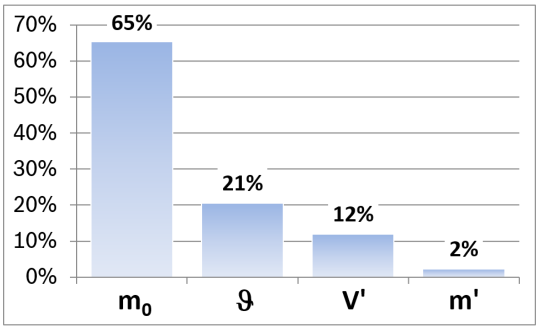

Figure 25: Pareto chart of the uncertainty contributions $(c_i \cdot u_i)^2$ to the standard uncertainty of m

Expanded measurement uncertainty

The expanded measurement uncertainty U_m is calculated with $k_p = 2$:

$$U_m = k_p \cdot u_m = 2 \cdot 0.113789\ \text{g} = 0.227578\ \text{g} \approx 0.228\ \text{g}$$

NOTE: The expanded measurement uncertainty of the output quantity is based, among others, on an input quantity which is determined from n_M = 5 measurement results only (ν = 4 degrees of freedom). According to appendix D.3, it should be checked in such cases whether the effective number of degrees of freedom ν_{eff} of the output quantity still reaches an order of magnitude of at least 15 … 20. Otherwise a higher coverage factor k_p should be used which is properly adjusted to ν_{eff}. Assuming that the uncertainty data for the EMI chamber volume and the scale indication can be considered to be secured at a maximum of 80%, 27 effective degrees of freedom result, i.e. k_p = 2.097 at a confidence level of 95.45%. k_p = 2 instead of 2.097 is usually considered to be acceptable. At a maximum of 75%, still 18 degrees of freedom result (k_p = 2.149).

Complete measurement result

For the adjusted EMI measuring instrument, the measurement data of the present case result in the following complete measurement result (applicable for each 1000 individual injection operations):

$$\overline{m} \pm U_m = 200.412\ \text{g} \pm 0.228\ \text{g}$$

This means that the conventional value of the measurement result can be expected in the range (200.412 ± 0.228) g with a confidence level of 95.45%, i.e. between 200.184 g and 200.640 g.

J.7.2 Calibration of the EMI measuring instrument

Measurement results

Measurement no.		1	2	3	4	5	Mean value	Standard deviation
EMI injection mass	m / g	200.47	200.47	200.46	200.51	200.53	200.488	0.030332
Scale injection mass	m_0 / g	200.47	200.49	200.48	200.49	200.51	200.488	0.014832
Difference	Δm / g	0.00	– 0.02	– 0.02	0.02	0.02	0.0	0.02

Table 15: Calibration of EMI, injection mass indicated by EMI and scale
(Mass of 1000 individual injection operations added up in each case)

Uncertainty of the deviation |m – m_0| between the indications of EMI and scale

The measuring process in the calibration laboratory does not reveal any deviation of the adjusted EMI from the standard device (scale) for a mean injection mass of 200.488 g, i.e. the mean deviation of 5 measurement series is zero (see Table 15).

Measurement results are considered to be different at a specific confidence level (e.g. 95.45% in case of k_p = 2) if their uncertainty ranges do not overlap (cf. chapter 2.2), i.e. if the condition $m + U_m < m_0 - U_0$ is met in the case $m < m_0$, or $m_0 + U_0 < m - U_m$ in the case $m_0 < m$, or generally if the absolute value of the difference of the measurement results is greater than the sum of their uncertainties:

$$\frac{|m - m_0|}{U_m + U_0} > 1$$

Because of $|\overline{m} - \overline{m}_0| = 0$, this condition generally cannot be fulfilled in this case, i.e. the results for m and m_0 must be considered to be identical (in terms of the above criterion).

The same applies to the individual measurement series. The maximum difference of the results in Table 15 is

$$\frac{|m - m_0|}{U_m + U_0} = \frac{\mathrm{MAX}(|m_i - m_{0i}|)}{U_m + U_0} \approx \frac{0.02\,\mathrm{g}}{0.228\,\mathrm{g} + 0.184\,\mathrm{g}} \approx \frac{0.02}{0.412} \approx 0.049 < 1$$

NOTE: The same applies to the application of the (more critical) criterion according to appendix G:

$$\frac{|m - m_0|}{\sqrt{U_m{}^2 + U_0{}^2}} = \frac{\mathrm{MAX}(|m_i - m_{0i}|)}{\sqrt{U_m{}^2 + U_0{}^2}} \approx \frac{0.02\,\mathrm{g}}{\sqrt{0.228^2\,\mathrm{g}^2 + 0.184^2\,\mathrm{g}^2}} \approx \frac{0.02}{0.293} \approx 0.068 < 1$$

J.7.3 Transferability of the results

The determined measurement uncertainty applies to the measuring process in the calibration laboratory. It can be transferred directly to measuring processes in other measuring laboratories only if these processes are performed under identical conditions. This always involves the sum of $n = 1000$ injection operations to be determined and evaluated.

NOTE: In case of regarding a single injection operation, instead of the mean value dispersion of n_M = 5, measurement series each with 1000 injections, the individual value dispersion has to be used for the calculations which is larger by a factor of $\sqrt{1000}$.

In case the EMI measuring device is utilized as part of a complex measuring process which differs significantly from the EMI usage in the calibration laboratory, the results for the measurement uncertainty cannot be transferred directly. In this case, the uncertainty data given in the EMI calibration certificate has to be seen as a contribution to the measurement uncertainty of the complex overall process which has to be determined by means of an uncertainty study specially tailored to this process.

Seq. No.	Description	Variable (symbol)	Measuring unit	Value of the variable	Value of the uncertainty data	Comments (z.B. references, explanatory notes, links to documents)	Evaluation type	Type A: Number of measured values; Type B: kp (≥1), confidence level (%), distribution	Numerical factor for calculating the standard uncertainty	Standard uncertainty	Sensitivity coefficient	Contribution to uncertainty	Contribution to uncertainty (squared)	Percentage contribution to MU budget $\frac{(c_i \cdot u(x_i))^2}{\sum_{i=1}^{n}(c_i \cdot u(x_i))^2}$	Rank (according to Pareto)	Estimated uncertainty of the uncertainty data	Degrees of freedom	Contribution to the denominator of the Welch-Satterthwaite formula
	Information about input quantities						**Standard uncertainties of input quantities**				**Contributions to the measurement uncertainty of the measurand**					**Determining kp for the measurand**		
i				x_i	Δx_i		A B	m_i k_p, %, name	1 or $\frac{\sqrt{m_i}}{k_p}$	$u(x_i) = \Delta x_i / k_p$	c_i	$c_i * u(x_i)$	$(c_i * u(x_i))^2$	[%]		$\Delta u(x_i) / u(x_i)$	v_i	$\frac{(c_i \cdot u(x_i))^4}{v_i}$
1	EMI chamber temperature	ϑ	°C	67,376	0,500000	Manufacturer information about thermocouple calibration	B	Rectangular distribution	1,732051	0,288676	-0,178294	-0,051470	0,002649160900	20,46%	2		1E+99	0
2	EMI chamber volume (uncorrected)	V'	cm^3		0,100000	Manufacturer information about uncertainty of LVDT volume measurements	B	Normal distribution	2,000000	0,050000	0,787363	0,039369	0,001549918161	11,97%	3	20,0%	12	2,0019E-07
3	EMI indication (uncorrected)	m'	g		0,037683	Standard deviation of EMI indications	A	5	2,236068	0,016853	-1,000669	-0,016865	0,000284428225	2,20%	4		4	2,0225E-08
4	Scale indication (absolute value)	m_0	g		0,184000	Calibration certificate of the scale: $U(m_0)$ = 0,184g; k_p = 2	B	Normal distribution	2,000000	0,092000	1,000000	0,092000	0,008464000000	65,37%	1	20,0%	12	5,9699E-06
5	Scale indication (dispersion)	δm_0	g	0,000	0,039623	Standard deviation of scale indications is contained in s(m') and $U(m_0)$												
6																		
7																		
8																		
9																		
10																		
11																		

Model equation:

$$m = \rho(\vartheta,p) \cdot x \cdot \pi \cdot \left(\frac{d+k_f}{2}\right)^2 = \rho(\vartheta) \cdot V' \cdot \frac{m_0}{m'}$$

Total result:

u_c^2 =	0,012948	100,00%		$\sum_i$ =	6,19E-06
u_c =	0,113789			ν_{eff} =	27
k_p =	2,000			1 - α =	95,450%
U =	**0,228**			k_p =	2,097

Table 16: Uncertainty budget for the “EMI” example

J.8 Pressure sensor

A commercially available pressure sensor is calibrated by means of a pressure balance for immediate practical use. The corresponding measurement uncertainty is determined. In contrast to the so-called "calibration uncertainty" which is usually specified on calibration certificates and which only takes account of the uncertainties of the calibration in the calibration laboratory, the additional uncertainties during subsequent practical use of the sensor are also taken into account in this example. So, no additional measurement uncertainty study is required. Furthermore, the effects on the measurement uncertainty are quantified if the sensor is used outside the calibrated temperature range and if corrections are waived.

J.8.1 Calibration uncertainty of the pressure sensor

Description of the measurement

A Hottinger P3M pressure sensor is calibrated for the pressure range 0 bar $\leq p_N \leq$ 100 bar[27]. The cleaned pressure sensor (object of calibration, measuring object) is screwed onto the pressure balance (standard device). The nominal pressure p_N is produced via the piston surface by placing a combination of reference masses (see page 82, footnote 26) on the pressure balance .

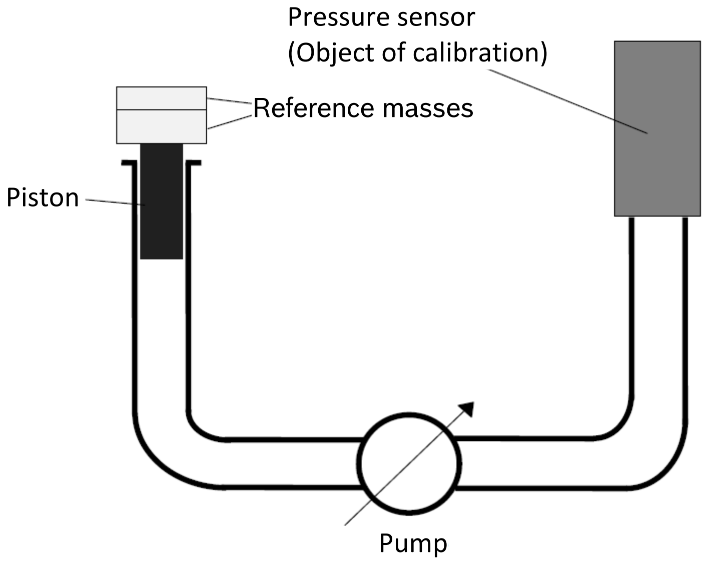

Figure 26: Measuring principle of a pressure balance with medium oil

Input quantities

- Information about the standard device [28] — Haenni ZP 36 pressure balance (JMM9Q003)

Uncertainty of the reference masses	$U_m = 0.0001\,\text{kg};\ k_p = 2$
Piston area at reference temperature ϑ_0	$A_0 = (0.040329 \pm 0.000018)\ \text{cm}^2;\ k_p = 2$
Reference temperature	$\vartheta_0 = 20\ °\text{C}$
Influence of temperature on the piston area:	
Volumetric thermal coefficient of expansion	$(\alpha + \beta) = 2.3 \cdot 10^{-5}\ \text{K}^{-1}$
Influence of deformation on the piston area:	
Deformation factor	$\lambda = (6.05 \pm 2.02) \cdot 10^{-7}\ \text{bar}^{-1};\ k_p = 2$
Local gravitational acceleration at the place of use [29]:	
(place where the pressure sensor is calibrated)	$g = 9.80852\ \text{ms}^{-2}$

[27] All pressures given are positive pressures with reference to atmospheric pressure
[28] See DAkkS calibration certificate for Haenni ZP 36
[29] According to data provided by Physikalisch-Technische Bundesanstalt (PTB)

The following reference masses are used for the calibration of the pressure sensor using the pressure balance:

Mass no.	Nominal pressure p_N / bar	Mass m / kg
7	40	1.6448
8	40	1.6449
9	20	0.8222
Piston (K)	20	0.8224

Table 17: Pressure sensor calibration, reference masses used

NOTE 1: The reference masses are marked with the nominal pressure p_N which is created when placed on the pressure balance. The corresponding effective masses m, which already take account of influences resulting from buoyancy and oil surface tension, are taken from the calibration certificate.

- Information about the calibration object (measuring object) [30]
  The deviation of the pressure p′ indicated by the sensor due to temperature influence amounts to a maximum of 0.1% per 10 K within the range –10°C to +80°C. The digit increment is 0.01 bar.
- Information about the procedure
  At different pressure settings, n = 3 repeated measurements are carried out in each case at ambient temperature $\vartheta = (23 \pm 0.1)$ °C. The pressures required in each case are obtained by placing appropriate combinations of reference masses on the pressure balance.

  EXAMPLE: The piston with applied mass no. 8 creates the nominal pressure $p_N = (20 + 40)$ bar = 60 bar.

Model

$$p = \underbrace{p' + K}_{=p_0} + \underbrace{\delta p_{Cal} + \delta p_m + \delta p_A + \delta p_\vartheta + \delta p_\lambda}_{=\delta p_0\ (\text{Standard})} + \underbrace{\delta K + \delta p_{\delta\vartheta} + \delta p_{\Delta\vartheta} + \delta p_{Res} + \delta p_{Hys} + \delta p_{Rpt}}_{=\delta p_S\ (\text{Sensor})}$$

with

p	corrected indication of the pressure sensor (calibration object, measuring object),
p'	uncorrected indication of the pressure sensor,
K	correction of the indication of the pressure sensor,
p_0	pressure (conventional value) created by the pressure balance (standard device),
δp_0	**deviations of the pressure created by the pressure balance due to …**
δp_{Cal}	… the limited accuracy of the calibration of the pressure balance,
δp_m	… the limited accuracy of the calibration of the reference masses,
δp_A	… the limited accuracy of the piston area,
δp_ϑ	… temperature fluctuations during sensor calibration,
δp_λ	… the limited accuracy of the piston deformation,
δp_S	**deviations of the pressure indicated by the pressure sensor due to …**
δK	… the limited accuracy of the correction of the indication,
$\delta p_{\delta\vartheta}$	… temperature fluctuations during sensor calibration,
$\delta p_{\Delta\vartheta}$	… deviating ambient temperature during sensor use,
δp_{Res}	… limited resolution,
δp_{Hys}	… hysteresis,
δp_{Rpt}	… the limited repeatability of a measurement result.

$-\Delta p \le \delta p \le \Delta p$ applies to all above-mentioned deviations δp. Here, δp describes the instantaneous value of the fluctuating deviation (expected value $\delta p = 0$), Δp the associated maximum deviation.

[30] See Hottinger P3M data sheet

Submodel for the pressure p_0 actually produced by the pressure balance at nominal pressure p_N

When the pressure balance is used as the standard device, the environmental conditions at the place of use must be taken into account, i.e. the effect of local gravitational acceleration g and ambient temperature ϑ as well as the effect of the reference mass m on the surface area and deformation of the piston and thereby on the generated pressure.

Pressure is defined as a force F per area A. The force F is defined as a mass m times acceleration. In case of weight forces the acceleration is given by the local gravitational acceleration g. Thus, the pressure generated by the pressure balance is calculated as

$$p_0 = \frac{F}{A} = \frac{m \cdot g}{A} \tag{J.14}$$

According to the calibration certificate, the area A is calculated using the following formula (see [EURAMET]):

$$A = A_0 \cdot \underbrace{\left(1 + \lambda \cdot p_0^*\right)}_{= f_\lambda} \cdot \underbrace{\left\{1 + (\alpha + \beta) \cdot (\vartheta - \vartheta_0)\right\}}_{= f_\vartheta} \tag{J.15}$$

with

A_0	piston area at reference temperature $\vartheta_0 = 20\ ^\circ\mathrm{C}$ and reference pressure $p = 0\ \mathrm{bar}$,
f_λ	correction factor: consideration of area changes due to piston deformation caused by applied reference masses,
λ	deformation factor,
p_0^*	generated pressure p_0 or approximated value [EURAMET],
f_ϑ	correction factor: consideration of deviations of the ambient temperature ϑ from the reference temperature ϑ_0,
$\alpha + \beta$	thermal coefficient of expansion,
ϑ	ambient temperature at the place of use of the pressure balance,
ϑ_0	reference temperature: ambient temperature at the place of calibration of the pressure balance.

Instead of p_0 the nominal value p_N is used as an approximated value for the pressure p_0^*:

$$p_0^* \approx p_N \tag{J.16}$$

The Eqs. (J.16) and (J.15) substituted in Eq. (J.14) yields

$$p_0 = \frac{m \cdot g}{A_0 \left(1 + \lambda \cdot p_N\right) \cdot \left\{1 + (\alpha + \beta) \cdot (\vartheta - \vartheta_0)\right\}} \tag{J.17}$$

NOTE 2: A prerequisite for meaningful results is that all parameters are included in the calculations with measurement units which are “compatible with each other”. If, for example, pressures given once in bar and once in N/m^2 are used in the same formula, the result may deviate from the correct result by several orders of magnitude. Therefore all input parameters should be converted into SI units (e.g. mbar or bar into N/m^2). In this example, areas are converted according to 1 cm^2 = 10^{-4} m^2 and pressure is converted according to 1 bar = 10^5 N/m^2 at 1 N = 1 kg·m/s^2.

NOTE 3: If, instead of Eq. (J.16), the model is derived based on $p_0^* = p_0$ with p_0 according to Eq. (J.14), Eq. (J.15) changes into a quadratic equation for the area A. Corresponding to the more complex solution for A, the model equation for p_0 becomes more complicated. The comparison of the calculated numerical values shows, however, that both variants of the model equation lead to the same results for all other calculations.

NOTE 4: For all calculations and particularly for those performed for comparison purposes, the present example turns out that rounding of intermediate results must be avoided as far as possible, since the numerical values of intermediate results can have very different orders of magnitude. If intermediate results cannot be avoided (e.g. in case of manual calculations), it is essential to avoid falling below a specific minimum number of significant digits in order to ensure a final result with reproducible numerical values. Particularly in the present example, intermediate results must not be rounded to <u>less than</u> 7 significant digits, i.e. in case of a decimal power representation with a so-called normalized mantissa, 1 pre-decimal position and 6 decimal places are required (such as $1.234567 \cdot 10^8$).

Measurement results

Repeated measurements at different nominal pressures p_N of the standard device result in the following indications p_S of pressure sensor:

Applied masses No.	Nominal pressure p_N / bar	Sensor indications Measurement series 1 p_S / bar	Sensor indications Measurement series 2 p_S / bar	Sensor indications Measurement series 3 p_S / bar	Mean value $\overline{p}_S$ / bar	Standard deviation s_S / bar
—	0	0.00	0.00	-0.02	-0.007	0.012
Piston (K)	20	20.02	20.02	20.01	20.017	0.006
K + 9	40	40.03	40.03	40.01	40.023	0.012
K + 8	60	60.09	60.09	60.09	60.090	0.000
K + 8 + 9	80	80.03	80.03	80.03	80.030	0.000
K + 7 + 8	100	99.95	99.95	99.94	99.947	0.006
K + 7 + 8	100	99.95	99.95	99.94	99.947	0.006
K + 8 + 9	80	80.09	80.08	80.07	80.080	0.012
K + 8	60	60.15	60.16	60.16	60.157	0.006
K + 9	40	40.08	40.07	40.08	40.077	0.006
Piston (K)	20	20.05	20.06	20.05	20.053	0.006
—	0	0.00	-0.02	0.00	-0.007	0.012

Table 18: Pressure sensor calibration, values indicated by the sensor

The mean values $\overline{p}_S$ are considered to be the uncorrected measurement results p': $p' = \overline{p}_S$.

Correction

- **Pressure p_0 of the pressure balance actually generated at nominal pressure p_N**

  According to Eq. (J.17) the following pressure is actually generated for a nominal pressure of e.g. $p_N = 100\text{ bar}$:

$$p_0 = \frac{4.1121\,\text{kg} \cdot 9.80852\,\frac{\text{m}}{\text{s}^2}}{0.040329 \cdot 10^{-4}\,\text{m}^2 \cdot \left\{1 + 6.05 \cdot 10^{-7}\,\frac{1}{10^5\,\frac{\text{N}}{\text{m}^2}} \cdot 100 \cdot 10^5\,\frac{\text{N}}{\text{m}^2}\right\} \cdot \left\{1 + 2.3 \cdot 10^{-5}\,\frac{1}{\text{K}} \cdot (23 - 20)\,\text{K}\right\}}$$

$$= 99.9985 \cdot 10^5\,\frac{\text{N}}{\text{m}^2} = 99.9985\text{ bar}$$

NOTE 5: The applied mass $m = m_K + m_7 + m_8$ is calculated using the values according to Table 17.

The same calculation carried out for all relevant mass combinations yields:

Applied masses No.	Nominal pressure p_N / bar	Generated pressure p_0 / bar
—	0	0.0000
Piston (K)	20	20.0002
K + 9	40	39.9950
K + 8	60	60.0015
K + 8 + 9	80	79.9954
K + 7 + 8	100	99.9985

Table 19: Pressure sensor calibration, pressure effective at the place of sensor calibration

- **Determination of the corrections required for the indications of the pressure sensor**

During calibration, the difference $\Delta p = \overline{p}_S - p_0$ exists between the actually effective pressure of the pressure balance and the indication of the pressure sensor to be calibrated.

Applied masses No.	Nominal pressure p_N / bar	Generated pressure p_0 / bar	Indicated pressure $p' = \overline{p}_S$ / bar	Deviation Δp / bar	Mean indication $\overline{\overline{p}}_S$ / bar	Mean deviation $\overline{\Delta p}$ / bar
—	0	0.0000	-0.007	-0.007	-0.007	-0.007
Piston (K)	20	20.0002	20.017	0.017	20.035	0.035
K + 9	40	39.9950	40.023	0.028	40.050	0.055
K + 8	60	60.0015	60.090	0.089	60.124	0.122
K + 8 + 9	80	79.9954	80.030	0.035	80.055	0.060
K + 7 + 8	100	99.9985	99.947	-0.052	99.947	-0.052
K + 7 + 8	100	99.9985	99.947	-0.052	99.947	-0.052
K + 8 + 9	80	79.9954	80.080	0.085	80.055	0.060
K + 8	60	60.0015	60.157	0.155	60.124	0.122
K + 9	40	39.9950	40.077	0.082	40.050	0.055
Piston (K)	20	20.0002	20.053	0.053	20.035	0.035
—	0	0.0000	-0.007	-0.007	-0.007	-0.007

Table 20: Pressure sensor calibration, generated and indicated pressure

The determined deviations $\Delta p = \overline{p}_S - p_0$ plotted versus the pressure values $\overline{p}_S$ indicated by the pressure sensor yields a so-called deviation chart (Figure 27).

In order to estimate the correction K, the mean values $\overline{\Delta p}$ of the deviations Δp which correspond to each other at increasing and decreasing pressure are calculated for each nominal pressure p_N. The same approach is used for the mean values $\overline{\overline{p}}_S$ of the indications $\overline{p}_S$ (see Figure 27, dashed line).

Then, the correction chart is represented by a graphically approximated curve or a mathematically determined regression curve which is fitted to the mean values $\overline{\Delta p}$ with opposite sign (Figure 28). In the present example, the correction curve is approximated by a regression using a third-order polynomial:

$$K\left(\overline{\overline{p}}_S\right) = a_0 + a_1 \cdot \overline{\overline{p}}_S + a_2 \cdot \overline{\overline{p}}_S{}^2 + a_3 \cdot \overline{\overline{p}}_S{}^3 \tag{J.18}$$

with $a_0 = 5.3973 \cdot 10^{-3}$ bar, $a_1 = -5.3202 \cdot 10^{-4}$, $a_2 = -6.7279 \cdot 10^{-5}\ \text{bar}^{-1}$, $a_3 = 7.7499 \cdot 10^{-7}\ \text{bar}^{-2}$.

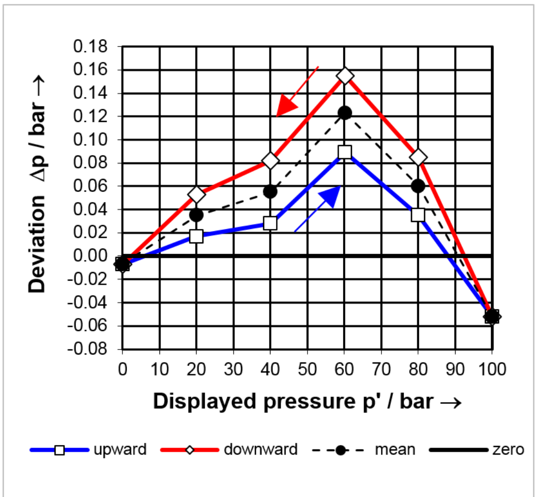

Figure 27: Deviation chart

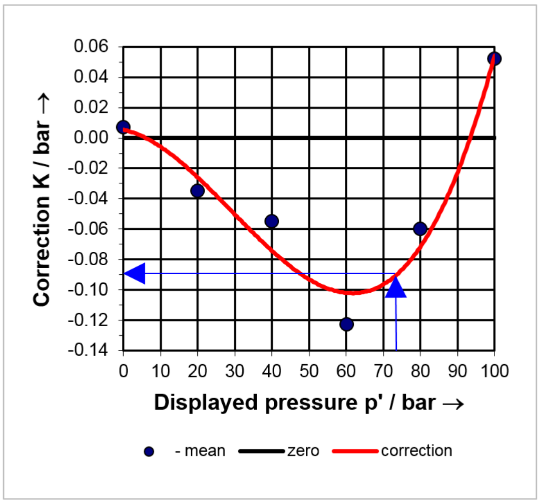

Figure 28: Correction chart

- **Correction of the pressure sensor indications**

  A pressure p′ indicated by the pressure sensor is corrected by the associated correction value $K(p')$ which is read from the correction chart or appropriately calculated and added to p':

  $$p_0 = p' + K(p')$$

  EXAMPLE: The pressure read on the pressure sensor is p' = 72 bar. The correction chart Figure 28 provides the correction K = –0.09 bar. So the correct pressure value is:
  p_0 = p' + K = 72 bar + (-0.09) bar = 71.91 bar.

  NOTE: The correction K includes an uncertainty δK which is exclusively caused by the regression. This uncertainty must be taken into account as an input quantity of the measurement uncertainty model, i.e. in addition to the uncertainties resulting from hysteresis, repeatability, etc.

<u>**Standard uncertainties of the input quantities**</u>

The majority of the determined standard uncertainties depends on the current pressure, i.e. the applied reference masses. Therefore the contribution of a certain input quantity to the overall uncertainty is estimated by means of the maximum standard uncertainty resulting from various mass combinations.

- **Standard device: Standard uncertainty u_{Cal} of the pressure balance resulting from the traceability to higher-level standards**

  Using the formula specified on the calibration certificate, the expanded measurement uncertainty U_{Cal} of the pressure balance is calculated for a certain pressure p_0 and converted to a standard uncertainty assuming a normal distribution according to

  $$u_{Cal} = \frac{U_{Cal}}{k_p} = \frac{1}{2}\sqrt{4{,}9 \cdot 10^{-5}\ \text{bar}^2 + 1{,}6 \cdot 10^{-7} \cdot {p_0}^2 + 4{,}1 \cdot 10^{-14}\ \text{bar}^{-2} \cdot {p_0}^4}$$

  The first summand of the radicand takes account of the uncertainty of the DAkkS reference standard [31]. The second summand takes account of the measurement uncertainty of the DAkkS working standard compared to the DAkkS reference standard. The third summand takes account of the deformation of the piston of the DAkkS standard [32]. The coverage factor is specified in the DAkkS calibration certificate as $k_p = 2$.

  The pressure $p_0 = 99{,}9985\ \text{bar}$, for example, results in the standard uncertainty

  $$u_{Cal} = \frac{1}{2}\sqrt{4{,}9 \cdot 10^{-5}\ \text{bar}^2 + 1{,}6 \cdot 10^{-7} \cdot (99{,}9985\ \text{bar})^2 + 4{,}1 \cdot 10^{-14}\ \text{bar}^{-2} \cdot (99{,}9985\ \text{bar})^4} \approx 0{,}0203\ \text{bar}$$

  This calculation is repeated for each pressure p_0 shown in Table 19. Finally the maximum uncertainty resulting from these calculations is used: $u_{Cal} = 0{,}0203\ \text{bar}$.

In addition to uncertainties of higher-level standards that are “inherited” as a result of traceability, uncertainties of the calibration of the reference masses, the piston area and the piston deformation must be taken into account as well as differences in the in environmental conditions between the place of use and the place of calibration of the standard device. These include e.g. different gravitational acceleration, different temperature and temperature fluctuations at the place of use.

The pressure p_0 which is actually effective at the place of use is described by Eq. (J.17). This equation represents a submodel which describes the pressure p_0 generated by the standard device at the place of use as a function of the applied mass m, the piston area A_0 in the calibration laboratory, the temperature ϑ at the place of use and the deformation coefficient λ. The uncertainties of these parameters are documented on the calibration certificate (except for ϑ).

[31] The DAkkS reference standard is the PTB national standard

[32] See also DAkkS calibration certificate

The uncertainty contributions to p_0 are determined by converting the maximum deviation of an input quantity (m, A_0, ϑ, λ) by means of the model equation into the corresponding deviation of the output quantity (p_0). If the deviation of the input quantity is not immediately known, the expanded uncertainty U is used instead.

NOTE 5: If it is assumed that the uncertainty U, specified on the calibration certificate at k_p = 2, was determined from the limit values a_+ and a_- assuming a normal distribution and a confidence level of 95% (cf. chapter 4.4.2.2), U corresponds to the maximum deviation Δa from the mean value a of the two limit values:

$$\frac{(a_+ - a_-)}{2} = \frac{(a+\Delta a)-(a-\Delta a)}{2} = \Delta a \qquad u = \frac{\Delta a}{2} \qquad U = 2\cdot u = 2\cdot\frac{\Delta a}{2} = \Delta a$$

NOTE 6: For models which are described by means of a single analytical equation such as Eq. (J.17), the uncertainties preferably should be calculated using sensitivity coefficients (see [GUM] or chap. 4.3.4). However, in order to avoid the required differentiations, the above briefly outlined calculation method is often applied. This method leads to identical results if the model behaves sufficiently linearly within the range of the associated uncertainties (i.e. approximation by a straight line whereby the proof of which is mathematically more challenging and outside the scope of booklet 8). This requirement is met for all model variants which could be used in the present example.

- **Standard device: Standard uncertainty u_m due to the uncertainty U_m of the reference masses m_k**

  The uncertainty of each individual mass m_k is specified on the calibration certificate (independently of the value m_k) with $U_m = 0{,}0001\,\text{kg}$ and coverage factor $k_p = 2$. Thus, in case of n_m applied masses m_k the following applies to the uncertainty:

$$\underbrace{\sqrt{U_m{}^2 + U_m{}^2 + U_m{}^2 + \ldots + U_m{}^2}}_{n_m\text{ terms}} = \sqrt{n_m \cdot U_m{}^2} = \sqrt{n_m}\cdot U_m$$

  The limit values of p_0 in terms of the total mass m of the applied reference masses are determined by using the extreme values $m + \sqrt{n_m}\cdot U_m$ and $m - \sqrt{n_m}\cdot U_m$ instead of m in Eq. (J.17):

$$p_0{}^{(+)} = \frac{\left(m + \sqrt{n_m}\cdot U_m\right)\cdot g}{A_0(1+\lambda\cdot p_N)\cdot\{1+(\alpha+\beta)\cdot(\vartheta-\vartheta_0)\}} \qquad p_0{}^{(-)} = \frac{\left(m - \sqrt{n_m}\cdot U_m\right)\cdot g}{A_0(1+\lambda\cdot p_N)\cdot\{1+(\alpha+\beta)\cdot(\vartheta-\vartheta_0)\}}$$

  The limit values $p_0{}^{(+)}$ and $p_0{}^{(-)}$ are used to determine the standard uncertainty of the output quantity p_0 caused by the uncertainty of the reference masses m according to chap. 4.4.2.2 assuming a normal distribution:

$$\Delta p_0 = \frac{\left|p_0{}^{(+)} - p_0{}^{(-)}\right|}{2} \qquad u_m = \frac{1}{2}\Delta p_0 = \frac{\left|p_0{}^{(+)} - p_0{}^{(-)}\right|}{4}$$

  Example: The nominal pressure $p_N = 100\,\text{bar}$, i.e. $n_m = 3$ applied masses with the total mass $m = m_K + m_7 + m_8 = 4{,}1121\,\text{kg}$ (see Table 17), results in the limit values

$$p_0{}^{(+)} = \frac{\left(4.1121 + \sqrt{3}\cdot 0.0001\right)\text{kg}\cdot 9.80852\,\frac{\text{m}}{\text{s}^2}}{0.040329\cdot 10^{-4}\,\text{m}^2\cdot\left(1 + 6.05\cdot 10^{-7}\,\frac{1}{10^5\,\frac{\text{N}}{\text{m}^2}}\cdot 100\cdot 10^5\,\frac{\text{N}}{\text{m}^2}\right)\cdot\left\{1 + 2.30\cdot 10^{-5}\,\frac{1}{\text{K}}\cdot(23-20)\text{K}\right\}}$$

$$\approx 100.002705\cdot 10^5\,\frac{\text{N}}{\text{m}^2} = 100.002705\ \text{bar}$$

  and

$$p_0^{(-)} = \frac{\left(4.1121\,\mathrm{kg} - \sqrt{3}\cdot 0.0001\,\mathrm{kg}\right)\cdot 9.80852\,\frac{\mathrm{m}}{\mathrm{s}^2}}{0.040329\cdot 10^{-4}\,\mathrm{m}^2 \cdot \left(1 + 6.05\cdot 10^{-7}\,\frac{1}{10^5\,\frac{\mathrm{N}}{\mathrm{m}^2}}\cdot 100\cdot 10^5\,\frac{\mathrm{N}}{\mathrm{m}^2}\right)\cdot\left\{1 + 2.30\cdot 10^{-5}\,\frac{1}{\mathrm{K}}\cdot (23-20)\,\mathrm{K}\right\}}$$

$$\approx 99.994281\cdot 10^5\,\frac{\mathrm{N}}{\mathrm{m}^2} = 99.994281\,\mathrm{bar}$$

and the standard uncertainty

$$u_m = \frac{100.002705\,\mathrm{bar} - 99.994281\,\mathrm{bar}}{4} = 0.002106\,\mathrm{bar}$$

This calculation is performed for all mass combinations used.
Resulting maximum uncertainty: $\left|u_m\right| = 0{,}002106\,\mathrm{bar}$.

- **Standard: Standard uncertainty u_A due to the uncertainty U_A of the piston area A_0**

$$U_A = 0{,}000018\,\mathrm{cm}^2 = 0{,}000018\cdot 10^{-4}\,\mathrm{m}^2$$

The extreme values $A_0 + U_A$ and $A_0 + U_A$ are used instead of A_0 in Eq. (J.17):

$$p_0^{(+)} = \frac{m\cdot g}{(A_0 + U_A)\cdot(1+\lambda\cdot p_N)\cdot\{1+(\alpha+\beta)\cdot(\vartheta-\vartheta_0)\}} \qquad p_0^{(-)} = \frac{m\cdot g}{(A_0 - U_A)\cdot(1+\lambda\cdot p_N)\cdot\{1+(\alpha+\beta)\cdot(\vartheta-\vartheta_0)\}}$$

Resulting maximum uncertainty: $\left|u_A\right| = 0{,}022316\,\mathrm{bar}$.

- **Standard device: Standard uncertainty u_ϑ due to temperature fluctuations within the $\pm\Delta\vartheta$ range during the measurement**

$$\Delta\vartheta = 0{,}1°\mathrm{C}$$

The extreme values $\vartheta + \Delta\vartheta$ and $\vartheta - \Delta\vartheta$ are used instead of ϑ in Eq. (J.17):

$$p_0^{(+)} = \frac{m\cdot g}{A_0\cdot(1+\lambda\cdot p_N)\cdot\{1+(\alpha+\beta)\cdot(\vartheta+\Delta\vartheta-\vartheta_0)\}} \qquad p_0^{(-)} = \frac{m\cdot g}{A_0\cdot(1+\lambda\cdot p_N)\cdot\{1+(\alpha+\beta)\cdot(\vartheta-\Delta\vartheta-\vartheta_0)\}}$$

Resulting maximum uncertainty: $\left|u_\vartheta\right| = 0{,}000115\,\mathrm{bar}$.

- **Standard: Standard uncertainty u_λ due to the uncertainty U_λ of the deformation factor λ**

$$U_\lambda = 2{,}02\cdot 10^{-7}\,\frac{1}{\mathrm{bar}} = 2{,}02\cdot 10^{-7}\,\frac{1}{10^5\,\frac{\mathrm{N}}{\mathrm{m}^2}}$$

The extreme values $\lambda + U_\lambda$ and $\lambda - U_\lambda$ are used instead of λ in Eq. (J.17):

$$p_0^{(+)} = \frac{m\cdot g}{A_0\cdot\{1+(\lambda+U_\lambda)\cdot p_N\}\cdot\{1+(\alpha+\beta)\cdot(\vartheta-\vartheta_0)\}} \qquad p_0^{(-)} = \frac{m\cdot g}{A_0\cdot\{1+(\lambda-U_\lambda)\cdot p_N\}\cdot\{1+(\alpha+\beta)\cdot(\vartheta-\vartheta_0)\}}$$

Resulting maximum uncertainty: $\left|u_\lambda\right| = 0{,}001010\,\mathrm{bar}$.

- **Object to be calibrated (measuring object): Standard uncertainty u_K of the correction**

  The uncertainty is estimated using the difference $-\overline{\Delta p}-K$ between the deviations $-\overline{\Delta p}$ of the mean values of the pressure sensor indications for each pressure $\overline{\overline{p}}_S$ and the corresponding values $K\left(\overline{\overline{p}}_S\right)$ of the regression curve:

Nominal pressure p_N / bar	Deviation $-\overline{\Delta p}$ / bar	Regression K / bar	Difference $-\overline{\Delta p}-K$ / bar
0	0.007	0.0054	0.0016
20	-0.035	-0.0260	-0.0090
40	-0.055	-0.0744	0.0194
60	-0.122	-0.1020	-0.0210
80	-0.060	-0.0713	0.0113
100	0.052	0.0544	-0.0024

Table 21: Difference between the established deviation and the calculated correction

The causes of the differences are not analyzed and the differences are therefore directly regarded as uncertainties (i.e. used without change): $u_K = -\overline{\Delta p} - K$.

Largest occurring uncertainty: $\left|u_K\right| = 0.0210\text{ bar}$.

NOTE 6: A more accurate estimate, which <u>could</u> lead to an even lower uncertainty contribution, requires to consider the residual dispersion s_R in relation to the regression curve, the uncertainties of the regression coefficients and their correlations. This corresponds to a generalization of the approach according to appendix F.2 which is mathematically very challenging and outside the scope of booklet 8. In the present case, the greater value of $u_K \leq 0.0223$ bar results for the maximum uncertainty of the correction. This value particularly applies at the limits p_N = 0 bar and p_N = 100 bar whereas a minimum value of $u_K \geq 0.0143$ bar is reached in the intermediate range.

- **Object to be calibrated (measuring object): Standard uncertainty $u_{\delta\vartheta}$ due to temperature fluctuations**

  According to the manufacturer's data sheet of the pressure sensor, a temperature-induced deviation $\delta p_{\delta\vartheta}$ has to expected in the $-10\,°C \leq \vartheta \leq +80\,°C$ temperature range. $\delta p_{\delta\vartheta}$ can amount up to 0.1% of the indicated pressure p' for every 10 K deviation of the ambient temperature ϑ from the reference temperature ϑ_{Ref} :

$$\delta p_{\delta\vartheta} = \frac{\vartheta - \vartheta_{Ref}}{10\,K} \cdot 0.001 \cdot p' \tag{J.19}$$

During calibration, temperature fluctuations occur up to a maximum of

$\Delta\vartheta = 0.1\,K$

The reference temperature ϑ_{Ref} is the nominal temperature ϑ during calibration of the pressure sensor ($\vartheta_{Ref} = \vartheta$). The instantaneous ambient temperature can deviate by a maximum of $\pm\Delta\vartheta$ ($\vartheta \pm \Delta\vartheta$). Therefore, the following applies to the maximum deviations of the pressure sensor indication p' :

$$\Delta p_{\delta\vartheta}^{(+)} = \frac{\Delta\vartheta}{10\,K} \cdot 0.001 \cdot p' \qquad \text{and} \qquad \Delta p_{\delta\vartheta}^{(-)} = \frac{-\Delta\vartheta}{10\,K} \cdot 0.001 \cdot p'$$

The standard uncertainty is calculated from these limit values assuming a normal distribution and a confidence level of 95% (cf. chapter 4.4.2.2):

$$u_{\delta\vartheta} = \frac{1}{2} \cdot \frac{\Delta p_{\delta\vartheta}^{(+)} - \Delta p_{\delta\vartheta}^{(-)}}{2} = \frac{1}{2} \cdot \frac{\Delta\vartheta}{10\,K} \cdot 0.001 \cdot p' \tag{J.20}$$

$\Delta\vartheta = 0.1\,K$ and $p' = 100\text{ bar}$ result in the maximum uncertainty contribution:

$$u_{\delta\vartheta} = \frac{1}{2} \cdot \frac{0.1\,K}{10\,K} \cdot 0.001 \cdot 100\,\text{bar} = 0.0005\,\text{bar}$$

- **Object to be calibrated (measuring object): Standard uncertainty $u_{\Delta\vartheta}$ due to temperature deviation**

  The pressure sensor is intended to be calibrated for practical us within the $(20 \pm 10)\,°\text{C}$ temperature range. At ambient temperatures of ϑ, which deviate from the reference temperature ϑ_{Ref}, deviations of the pressure sensor indications p' have be expected according to Eq. (J.19).

  If, during practical use, the actual ambient temperature ϑ is not taken into account during the measurement (i.e. there is no temperature correction), the maximum deviation from the reference temperature $\vartheta_{\text{Ref}} = 23\,°\text{C}$ (i.e. the temperature during sensor calibration) must be applied which is possible within the $(20 \pm 10)\,°\text{C}$ temperature range:

  $$\Delta\vartheta = \left|10\,°\text{C} - 23\,°\text{C}\right|$$

  The calculation is performed according to Eq. (J.20). $p' = 100\,\text{bar}$ results in the maximum uncertainty contribution:

  $$u_{\Delta\vartheta} = \frac{1}{2} \cdot \frac{\left|10\,°\text{C} - 23\,°\text{C}\right|}{10\,\text{K}} \cdot 0.001 \cdot 100\,\text{bar} = 0.065\,\text{bar}$$

  NOTE 7: The uncertainty of the reference temperature ϑ_{Ref} was considered in the previous section.

- **Object to be calibrated (measuring object): Standard uncertainty u_{Res} due to limited resolution**

  The influence of the resolution is contained in the standard deviation s_S of the pressure sensor indications p' (see Table 18). So it must not be considered separately:

  $$u_{\text{Res}} = 0\,\text{bar}$$

- **Object to be calibrated (measuring object): Standard uncertainty u_{Hys} due to pressure sensor hysteresis**

  Usually no special procedure is prescribed when using the pressure sensor, so that the hysteresis is not balanced and must be taken into account as an uncertainty. The values according to Table 18 result in the following differences of indications (hysteresis):

Nominal pressure p_N / bar	p_N rising: indications $\overline{p}_S(\uparrow)$ / bar	p_N falling: indications $\overline{p}_S(\downarrow)$ / bar	Difference of indications $\overline{p}_S(\downarrow) - \overline{p}_S(\uparrow)$ / bar
0	-0.007	-0.007	0
20	20.017	20.053	0.036
40	40.023	40.077	0.054
60	60.090	60.157	0.067
80	80.030	80.080	0.050
100	99.947	99.947	0

  Table 22: Pressure sensor calibration, hysteresis

  Table 22 shows a maximum hysteresis of 0.067 bar. The assumption of a U-shaped distribution with a span $\overline{p}_S(\downarrow) - \overline{p}_S(\uparrow)$ = 0.067 bar results in the maximum standard uncertainty:

  $$u_{\text{Hys}} = \frac{1}{\sqrt{2}} \frac{\overline{p}_S(\downarrow) - \overline{p}_S(\uparrow)}{2} \cdot \approx \frac{0.067\,\text{bar}}{1.414 \cdot 2} = 0.024\,\text{bar}$$

- **Procedure: Standard uncertainty u_{Rpt} of the repeatability of the measurement result**

  The maximum standard deviation s_S of the pressure sensor indications p_S is $s_S = 0{,}012\ \text{bar}$ (see Table 18). With $n = 3$ measured values and assuming a normal distribution the standard uncertainty is

  $$u_{Rpt} = \frac{s_S}{\sqrt{n}} = \frac{0.012\ \text{bar}}{\sqrt{3}} \approx 0.007\ \text{bar}$$

<u>Combined standard uncertainty of the output quantity</u>

The standard uncertainty u_C is calculated as

$$u_C = \sqrt{u_{Cal}^2 + u_m^2 + u_A^2 + u_\vartheta^2 + u_\lambda^2 + u_K^2 + u_{\delta\vartheta}^2 + u_{\Delta\vartheta}^2 + u_{Res}^2 + u_{Hys}^2 + u_{Rpt}^2}$$

$$= \sqrt{\begin{aligned}&(0.0203\ \text{bar})^2 + (0.002106\ \text{bar})^2 + (0.022316\ \text{bar})^2 + (0.000115\ \text{bar})^2 + (0.001010\ \text{bar})^2\\ &+ (0.0210\ \text{bar})^2 + (0.0005\ \text{bar})^2 + (0.065\ \text{bar})^2 + (0.000\ \text{bar})^2 + (0.024\ \text{bar})^2 + (0.007\ \text{bar})^2\end{aligned}}$$

$$\approx \sqrt{0.006207}\ \text{bar} \approx 0.079\ \text{bar}$$

The Pareto chart (Figure 29) of the individual uncertainty contributions u_i^2 shows that deviations of the ambient temperature during sensor use from the temperature during sensor calibration provide the main contribution to overall uncertainty. This contribution could be reduced significantly by temperature correction.

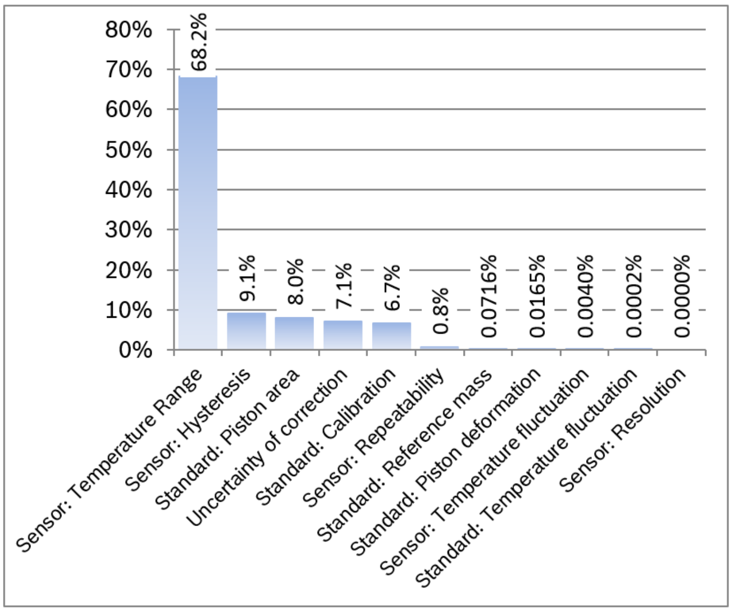

Figure 29: Pressure sensor; Pareto chart of the uncertainty contributions u_i^2

<u>Expanded measurement uncertainty</u>

The coverage factor $k_p = 2$ gives the expanded measurement uncertainty

$$U = k_p \cdot u_C \approx 2 \cdot 0.079\ \text{bar} = 0.158\ \text{bar}$$

<u>Complete measurement result</u>

The following complete measurement result applies for the pressure sensor in the pressure range $0\ \text{bar} \le p_N \le 100\ \text{bar}$ when used in the temperature range $10\ °C \le \vartheta \le 30\ °C$:

$$p = p_0 \pm 0.158\ \text{bar} = p' + K(p') \pm 0.158\ \text{bar}$$

Information about input quantities							Standard uncertainties of input quantities				Contributions to the measurement uncertainty of the measurand				
Seq. No.	Description	Variable (symbol)	Measuring unit	Value of the variable	Value of the uncertainty data	Comments (z.B. references, explanatory notes, links to documents)	Evaluation type	Type A: Number of measured values; Type B: kp (≥1), confidence level (%), distribution	Numerical factor for calculating the standard uncertainty	Standard uncertainty	Sensitivity coefficient	Contribution to uncertainty	Contribution to uncertainty (squared)	Percentage contribution to MU budget $\frac{(c_i \cdot u(x_i))^2}{\sum_{i=1}^{n}(c_i \cdot u(x_i))^2}$	Rank (according to Pareto)
i				x_i	Δx_i		A B	m_i k_p, %, name	1 or $\sqrt{m_i}$ k_p	$u(x_i) = \Delta x_i / k_p$	c_i	$c_i * u(x_i)$	$(c_i * u(x_i))^2$	[%]	
1	Standard: Calibration	δp_{Cal}	bar	0	0,040658	See text for calculation	B	Normal distribution	2,000000	0,020329	1	0,020329	0,000413268	6,7%	5
2	Standard: Reference mass	δp_m	bar	0	0,004212	See text for calculation	B	Normal distribution	2,000000	0,002106	1	0,002106	0,000004435	0,0716%	7
3	Standard: Piston area	δp_{A0}	bar	0	0,044632	See text for calculation	B	Normal distribution	2,000000	0,022316	1	0,022316	0,000498004	8,0%	3
4	Standard: Temperature fluctuation	δp_{ϑ}	bar	0	0,000230	See text for calculation	B	Normal distribution	2,000000	0,000115	1	0,000115	0,000000013	0,0002%	10
5	Standard: Piston deformation	δp_{λ}	bar	0	0,002020	See text for calculation	B	Normal distribution	2,000000	0,001010	1	0,001010	0,000001020	0,0165%	8
6	Uncertainty of correction	δK	bar	0	0,021000	See text for calculation	A		1,000000	0,021000	1	0,021000	0,000441000	7,1%	4
7	Sensor: Temperature fluctuation	$\delta p_{\delta\vartheta}$	bar	0	0,001000	See text for calculation	B	Normal distribution	2,000000	0,000500	1	0,000500	0,000000250	0,0040%	9
8	Sensor: Temperature Range	$\delta p_{\Delta\vartheta}$	bar	0	0,130000	See text for calculation	B	Normal distribution	2,000000	0,065000	1	0,065000	0,004225000	68,2%	1
9	Sensor: Resolution	δp_{Res}	bar	0	Deviations contained in *"Sensor: Repeatability"* (δp_{Rpt})										
10	Sensor: Hysteresis	δp_{Hys}	bar	0	0,033500	See text for calculation	B	U-distribution	1,414214	0,023688	1	0,023688	0,000561121	9,1%	2
11	Sensor: Repeatability	δp_{Rpt}	bar	0	0,012000	See text for calculation	A	3	1,732051	0,006928	1	0,006928	0,000047997	0,8%	6
							Total result:					**u_c^2 =**	0,006192	100,000%	
												u_c =	0,079		
												k_p =	2,000		
												U =	**0,158**		

Model equation:

$$p = \underbrace{p' + K}_{=p_0} + \underbrace{\delta p_{Cal} + \delta p_m + \delta p_A + \delta p_{\vartheta} + \delta p_{\lambda}}_{=\delta p_0\ (\text{Standard})} + \underbrace{\delta K + \delta p_{\delta\vartheta} + \delta p_{\Delta\vartheta} + \delta p_{Res} + \delta p_{Hys} + \delta p_{Rpt}}_{=\delta p_S\ (\text{Sensor})}$$

Expected values: $\delta p = 0$ Deviations: $-\Delta p \le \delta p \le \Delta p$

Table 23: Uncertainty budget for the “pressure sensor” example

The correction $K(p')$ is taken from the chart in Figure 28 or calculated according to Eq. (J.18).

This result means that during practical use of the pressure sensor and an indication of e.g. $p' = 72\,\text{bar}$ the conventional value of the measurement result can be expected between $72\,\text{bar} - 0{,}09\,\text{bar} - 0{,}158\,\text{bar} \approx 71{,}75\,\text{bar}$ and $72\,\text{bar} - 0{,}09\,\text{bar} + 0{,}158\,\text{bar} \approx 72{,}07\,\text{bar}$ with a confidence level of 95%.

J.8.2 Potential further uncertainties when working with the pressure sensor

In practical use of the pressure sensor,

- the pressure-dependent correction of the indication is often skipped and
- the sensor is used within the temperature range specified by the manufacturer, but outside the calibrated temperature range.

The corresponding impact on the uncertainty of the measuring results of the pressure sensor has to be taken into account in addition.

NOTE: It is assumed that a negligible time drift of the sensor occurs (e.g. as a result of environmental impact or aging). Otherwise, either a corresponding consideration in the uncertainty budget or another appropriate measure is required (e.g. adjustment, replacement by a new sensor).

<u>Temperature range (20 ± 10) °C without correction of the pressure sensor indication p‘</u>

If the pressure sensor is used in the $(20 \pm 10)\,°\text{C}$ temperature range but no pressure-dependent correction $K(p')$ is performed, the maximum possible correction K within the pressure range $0\,\text{bar} \le p_N \le 100\,\text{bar}$ has to be added as an additional uncertainty component to the uncertainty budget (see appendix F.3):

$$U = k_p \cdot \sqrt{{u_C}^2 + K^2(p')}$$

The maximum required correction K_{MAX} within the pressure range $0\,\text{bar} \le p_N \le 100\,\text{bar}$ is provided as the extreme value of the correction curve $K(p')$ which is either read from the chart in Figure 28 in or calculated using Eq. (J.18) (zero point of the 1st derivative):

$$K_{MAX} = K(p' \approx 61.632\,\text{bar}) = -0.1022\,\text{bar}$$

Expanded measurement uncertainty:

$$U = 2 \cdot \sqrt{0.006207 + (-0.1022)^2}\,\text{bar} = 2 \cdot \sqrt{0.016655}\,\text{bar} \approx 2 \cdot 0.129\,\text{bar} = 0.258\,\text{bar}$$

Complete measurement result:

$$p = p' \pm 0.258\,\text{bar}$$

Accordingly, a measurement uncertainty applies to the sensor that is enlarged by the factor 1.7 unless the correction is performed. For an indication of e.g. $p' = 72\,\text{bar}$ the conventional value of measurement result is now expected with a confidence interval of 95% between $72\,\text{bar} - 0.258\,\text{bar} \approx 71.74\,\text{bar}$ and $72\,\text{bar} + 0.258\,\text{bar} \approx 72.26\,\text{bar}$, i.e. in case of this particular indication the skipped correction primarily affects the upper limit of uncertainty.

<u>Temperature range –10°C ≤ ϑ ≤ +80°C without correction of the pressure sensor indication p‘</u>

If the pressure sensor is used over the entire temperature range that is permitted according to manufacturer's specification and no corrections are performed (deviation of the sensor indication from the standard, deviation of the ambient temperature from the reference temperature in the calibration laboratory), the maximum values that are possible within the provided pressure range and temperature range must be used for the uncertainty contributions $K(p')$ and $u_{\Delta\vartheta}(p')$.

Within the $-10\,°C \le \vartheta \le 80\,°C$ range, $\vartheta = 80\,°C$ is the temperature with the maximum possible deviation $\Delta\vartheta$ from the ambient temperature $\vartheta_{Ref} = 23\,°C$ during sensor calibration:

$$\Delta\vartheta = \left|80\,°C - 23\,°C\right|$$

The calculation is performed according to Eq. (J.20). $p' = 100\text{ bar}$ results in the maximum uncertainty contribution

$$u_{\Delta\vartheta} = \frac{1}{2} \cdot \frac{80\,°C - 23\,°C}{10\text{ K}} \cdot 0.001 \cdot 100\text{ bar} = 0.285\text{ bar}$$

This value replaces the uncertainty contribution $u_{\Delta\vartheta}$ included in u_C which previously was taken into account for the temperature range $10\,°C \le \vartheta \le 30\,°C$.

The expanded measurement uncertainty is calculated accordingly:

$$U = k_p \cdot \sqrt{u_C{}^2 - u_{\Delta\vartheta}{}^2(10\,°C \le \vartheta \le 30\,°C) + u_{\Delta\vartheta}{}^2(-10\,°C \le \vartheta \le 80\,°C) + K^2(p'_S)}$$

$$U = 2 \cdot \sqrt{0.006207 - (0.065)^2 + (0.285)^2 + (-0.1022)^2}\text{ bar} = 2 \cdot \sqrt{0.093655}\text{ bar} \approx 2 \cdot 0.306\text{ bar} = 0.612\text{ bar}$$

Complete measurement result:

$$p = p' \pm 0.612\text{ bar}$$

According to this, a measurement uncertainty must be applied to the sensor that is enlarged by the factor 4 if there is no correction and if it is not ensured that the sensor will be used only within the calibrated temperature range $(20 \pm 10)\,°C$. For an indication of e.g. $p' = 72\text{ bar}$ the conventional value of the measurement result is now expected between $72\text{ bar} - 0.612\text{ bar} \approx 71.39\text{ bar}$ and $72\text{ bar} + 0.612\text{ bar} \approx 72.61\text{ bar}$ with a confidence level of 95%.

Conclusion

The results show that missing correction and using the sensor outside of the calibrated temperature range causes additional uncertainties which account for almost 98% of all uncertainty contributions u_i^2 to the overall uncertainty (Figure 30). Therefore, in the practical application of the pressure sensor, it must be decided depending on the measuring task and the specific requirements for the measurement results, whether additional effort for the correction is justifiable and the usage of the sensor can be confined to the calibrated temperature range, or whether another correction with regard to the temperature should be considered.

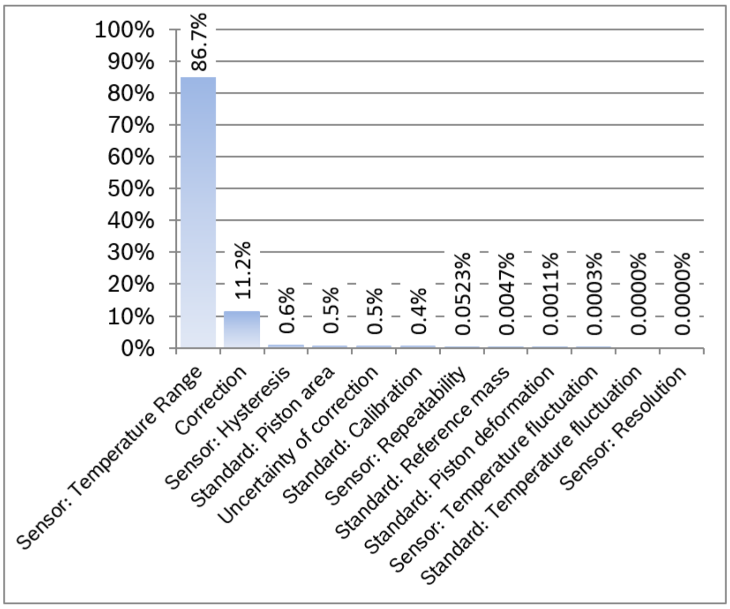

Figure 30: Pressure sensor; Pareto chart of the uncertainty contributions u_i^2 (no correction, $\vartheta \le 80°C$)

Table of Symbols

a	Half width of the interval between the limit values a_+ and a_-
a_+	Upper limit value (of a value distribution)
a_-	lower limit value (of a value distribution)
α_K	Intercept (correction) of the correction curve (calibration curve)
β_K	Slope (correction factor) of the correction curve (calibration curve)
c_i	Sensitivity coefficient assigned to the standard uncertainty of input quantity no. i
δx_i	Deviation of the value x_i from the conventional value of input quantity no. i
EV	**<u>E</u>**quipment **<u>V</u>**ariation, Repeatability
f	Model function
i	Index of (different) input quantities; $1 \le i \le n$
j	Index of data sets allocated to a (specific) input quantity; $1 \le j \le j_P$
j_P	Number of pooled data sets
k	Index of the values of a (specific) input quantity; $1 \le k \le m$
K	Correction (correction curve, calibration curve)
k_p	Coverage factor for the calculation of the expanded measurement uncertainty
m	Number of values assigned to a (specific) input quantity
n	Number of (different) input quantities
m_j	Number of values in data set no. j of a (specific) input quantitity
$r(x_i, x_j)$	Correlation coefficient of two data sets of the input quantities no. i and no. j
R	Resistance
$s(x_i)$	Standard deviation of the values x_{ik} of input quantity no. i
$s(x_i, x_j)$	Covariance of two data sets of the input quantities no. i and no. j
$s_j(x_i)$	Standard deviation of data set no. j of input quantity no. i
s_p	Pooled standard deviation
ϑ	Temperature in °C *(temperature differences in K)*
T	Tolerance of a measured characteristic
$u(\delta x_i)$	Standard uncertainty of the deviation of value x_i from the conventional value of input quantity no. i
$u(x_i)$	Standard uncertainty of input quantity no. i
$u(x_i, x_j)$	Covariance of the standard uncertainties of two data sets of the input quatities no. i and no. j
$u(\overline{x}_i)$	Standard uncertainty of the mean value of the values x_{ik} of the input quantity no. i
$u(\overline{x}_i, \overline{x}_j)$	Covariance of the standard uncertainties of the mean values of two data sets of the input quatities no. i and no. j
$u_C(y)$	Combined standard uncertainty of measurand y
U	Expanded measurement uncertainty
U_{cal}	Expanded uncertainty of calibration
U_{rel}	Expanded measurement uncertainty related to a reference value

x_i	Value of input quantity no. i
$\bar{x}_i$	Mean value of the values x_{ik} of the input quantity no. i
x_{ik}	Value no. k of the input quantity no. i
x_{ijk}	Value no. k in the data set no. j (e.g. measurement series) of input quantity no. i
x_m	Reference value of a reference / master (e.g. measuring standard, stability part)
y	Value of a measurand (output quantity, result)
y'	Uncorrected value of a measurand y ("raw value")
y_0	Conventional value of a measurand y (no uncertainty)

Further symbols which are used in individual chapters only are defined in the respective context..

Definition of terms

NOTE 1: The following definitions of terms were taken from the standards referenced in this document. Corresponding notes were only adopted in single cases if they were considered directly relevant and/or essential for understanding a standardized term. Otherwise, the respective standard should be referenced for notes and examples.

*NOTE 2: "Editorial notes" are **not** part of the respective standard.*

NOTE 3: The definitions of terms according to [VIM] were used preferably. If terms are not contained in [VIM], the most current definitions from [GUM] or the standards [ISO 3534-2], [ISO 3534-1], [ISO 9000], [ISO 14253], [DIN 1319-4] and [DIN 1319-1] were adopted (or listed additionally in some cases). Non-standardized definitions are only used if the listed standards do not provide a definition.

NOTE 4: Terms whose definitions are contained in the following summary are in bold if they are used in definitions of other terms.

calibration curve *(Ger. Kalibrierkurve)*
expression of the relation between **indication** and corresponding **measured quantity value**

*NOTE: A calibration curve expresses a one-to-one relation that does not supply a **measurement result** as it bears no information about the **measurement uncertainty**.*

[VIM, 4.31]

characteristic *(Ger. Merkmal)*
distinguishing feature

NOTE 1: A characteristic can be inherent or assigned.

NOTE 2: A characteristic can be qualitative or quantitative.

NOTE 3: There are various classes of characteristics such as the following:

- *physical (e.g. mechanical, electrical, chemical, biological);*
- *sensory (e.g. relating to smell, touch, taste, sight, hearing);*
- *behavioral (e.g. courtesy, honesty, veracity)*
- *temporal (e.g. punctuality, reliability, availability);*
- *ergonomic (e.g. physiological characteristic or related to human safety);*
- *functional (e.g. maximum speed of an aircraft).*

[ISO 3534-2, 1.1.1]

combined standard uncertainty *(Ger. Kombinierte Standardunsicherheit)*
standard measurement uncertainty that is obtained using the individual standard measurement uncertainties associated with the **input quantities** in a **measurement model** [VIM, 2.31]

confidence interval *(Ger. Vertrauensbereich)*
interval estimator (T_0, T_1) for the **parameter** θ with the **statistics** T_0 and T_1 as interval limits and for which it holds that $P[T_0 < \theta < T_1] \geq 1-\alpha$

NOTE 2: Associated with this confidence interval is the attendant performance characteristic 100·(1–α) %, where α is generally a small number. The performance characteristic, which is called the confidence coefficient or confidence level, is often 95 % or 99 %. The inequality P [T0 < θ < T1] $\geq$ 1 – α holds for any specific but unknown population value of θ.

[ISO 3534-1, 1.28]

EDITORIAL NOTE: P denotes a probability.

confidence level *(Ger. Vertrauensniveau)*
see **confidence interval**, note 2

conformity *(Ger. Konformität)*
Fulfilment of a **requirement** [ISO 9000, 3.6.11]

conformity evaluation *(Ger. Konformitätsbewertung)*
systematic examination of the extent to which an **item/entity** fulfils specified **requirements** [ISO 3534-2, 4.1.1]

conformity zone *(Ger. Konformitätsbereich)*
specification zone reduced by the **expanded measurement uncertainty** [ISO 14253-1, 3.20]

conventional (quantity) value *(Ger.vereinbarter Wert)*
quantity value attributed by agreement to a **quantity** for a given purpose

NOTE 1: The term "conventional true quantity value" is sometimes used for this concept, but its use is discouraged.

*NOTE 2: Sometimes a conventional quantity value is an **estimate** of a **true quantity value**.*

*NOTE 3: A conventional quantity value is generally accepted as being associated with a suitably small **measurement uncertainty**, which might be zero.*

[VIM, 2.12]

EDITORIAL NOTE: The term "conventional value" obviously replaces the normative term "conventional true value" according to [ISO 3534-2] which is no longer contained in the current release of [VIM].

conventional true value *(Ger. richtiger Wert)*
value of a quantity or quantitative **characteristic** which, for a given purpose, may be substituted for a **true value**

*NOTE 1: A conventional true value is, in general, regarded as sufficiently close to the **true value** for the difference to be insignificant for the given purpose.*

[ISO 3534-2, 3.2.6]

correction *(Ger. Korrektion)*
compensation for an estimated systematic effect

NOTE 1: See ISO/IEC Guide 98-3:2008, 3.2.3, for an explanation of 'systematic effect'.

NOTE 2: The compensation can take different forms, such as an addend or a factor, or can be deduced from a table.

[VIM, 2.53]

coverage factor *(Ger. Erweiterungsfaktor)*
number larger than one by which a **combined standard measurement uncertainty** is multiplied to obtain an **expanded measurement uncertainty** [VIM, 2.38]

degrees of freedom *(Ger. Freiheitsgrade)*
number of terms in a sum minus the number of constraints on the terms of the sum [ISO 3534-1, 2.54]

entity *(Ger. Einheit):* see **item** [ISO 3534-2, 1.2.11]

estimate *(Ger. Schätzwert)*
observed value of an **estimator** [ISO 3534-1, 1.31]

estimation *(Ger. Schätzung)*
procedure that obtains a statistical representation of a **population** from a **random sample** drawn from this population

*NOTE 1: In particular, the procedure involved in progressing from an **estimator** to a specific estimate constitutes **estimation**.*

[ISO 3534-1, 1.36]

estimator *(Ger. Schätzer)*
statistic used in **estimation** of the parameter Θ [ISO 3534-1, 1.12]

expanded measurement uncertainty *(Ger. Erweiterte Messunsicherheit)*
product of a **combined standard measurement uncertainty** and a factor larger than the number one

> *NOTE 2: The term "factor" in this definition refers to a **coverage factor**.*

[VIM, 2.35]

indicating measuring instrument *(Ger. anzeigendes Messgerät)*
measuring instrument providing an output signal carrying information about the **value** of the **quantity** being measured

> *NOTE 1: An indicating measuring instrument may provide a record of its **indication**.*
>
> *NOTE 2: An output signal may be presented in visual or acoustic form. It may also be transmitted to one or more other devices.*

[VIM, 3.3]

indication *(Ger. Anzeige)*
quantity value provided by a **measuring instrument** or a **measuring system** [VIM, 4.1]

influence quantity *(Ger. Einflussgröße)*
quantity that is not the **measurand** but that affects the **result of the measurement** [GUM, B.2.10; VIM(2), 2.7]

influence quantity *(Ger. Einflussgröße)*
quantity that, in a direct **measurement**, does not affect the quantity that is actually measured, but affects the relation between the **indication** and the **measurement result**

> *NOTE 2: In the GUM, the concept 'influence quantity' is defined as in the second edition of the VIM, covering not only the quantities affecting the measuring system, as in the definition above, but also those quantities that affect the quantities actually measured. Also, in the GUM this concept is not restricted to direct measurements.*

[VIM, 2.52]

input quantity (in a measurement model) *(Ger. Eingangsgröße)*
quantity that must be measured, or a quantity, the **value** of which can be otherwise obtained, in order to calculate a **measured quantity value** of a **measurand**. [VIM, 2.50]

inspection *(Ger. Prüfung)*
conformity evaluation by observation and judgement accompanied as appropriate by **measurement**, testing or gauging [ISO 3534-2, 4.1.2]

intermediate precision condition (of measurement) *(Ger. Vergleichbedingung)*
condition of **measurement**, out of a set of conditions that includes the same **measurement procedure**, same location, and replicate measurements on the same or similar objects over an extended period of time, but may include other conditions involving changes

> *NOTE 1: The changes can include new calibrations, calibrators, operators, and **measuring systems**.*

[VIM, 2.22]

item *(Ger. Einheit)*
anything that can be described and considered separately [ISO 3534-2, 1.2.11]

kind of quantity *(Ger. Art einer Größe, Größenart)*
aspect common to mutually comparable **quantities** [VIM, 1.2]

lower specification limit *(Ger. Mindestwert)*
specification limit that defines the lower limiting value [ISO 3534-2, 3.1.5]

material measure *(Ger. Maßverkörperung)*
measuring instrument reproducing or supplying, in a permanent manner during its use, **quantities** of one or more given **kinds**, each with an assigned **quantity value**

*NOTE 1: The **indication** of a material measure is its assigned **quantity value**.*

*NOTE 2: A material measure can be a **measurement standard**.*

[VIM, 3.6]

measurand *(Ger. Messgröße)*
quantity intended to be measured [VIM, 2.3]

measured (quantity) value *(Ger. Messwert)*
quantity value representing a **measurement result** [VIM, 2.10]

measurement *(Ger. Messung)*
process of experimentally obtaining one or more **quantity values** that can reasonably be attributed to a **quantity**

*NOTE 1: Measurement does not apply to **nominal properties**.*

NOTE 2: Measurement implies comparison of quantities and includes counting of entities.

*NOTE 3: Measurement presupposes a description of the quantity commensurate with the intended use of a **measurement result**, a **measurement procedure**, and a calibrated **measuring system** operating according to the specified measurement procedure, including the measurement conditions.*

[VIM, 2.1]

measurement error *(Ger. Messabweichung)*
measured quantity value minus a **reference quantity value** [VIM, 2.16]

measurement method *(Ger. Messmethode)*
generic description of a logical organization of operations used in a **measurement** [VIM, 2.5]

measurement model *(Ger. Modell der Messung)*
mathematical relation among all **quantities** known to be involved in a **measurement** [VIM, 2.48]

measurement principle *(Ger. Messprinzip)*
phenomenon serving as a basis of a **measurement** [VIM, 2.4]

measurement procedure *(Ger. Messverfahren)*
detailed description of a **measurement** according to one or more **measurement principles** and to a given **measurement method** based on a **measurement model** and including any calculation to obtain a **measurement result** [VIM, 2.6]

measurement process *(Ger. Messprozess)*
set of operations to determine the **value of a quantity** [ISO 9000, 3.11.5]

measurement result *(Ger. Messergebnis)*
set of **quantity values** being attributed to a **measurand** together with any other available relevant information [VIM, 2.9]

measurement standard *(Ger. Normal)*
realization of the definition of a given **quantity**, with stated **quantity value** and associated **measurement uncertainty**, used as a reference

> *NOTE 1: A "realization of the definition of a given quantity" can be provided by a **measuring system**, a **material measure**, or a reference material.*

[VIM, 5.1]

measurement uncertainty *(Ger. Messunsicherheit)*
non-negative parameter characterizing the dispersion of the **quantity values** being attributed to a **measurand**, based on the information used [VIM, 2.26]

measurement uncertainty *(Ger. Messunsicherheit)*
parameter, associated with the **result of a measurement**, that characterizes the dispersion of the values that could reasonably be attributed to the **measurand** [GUM, 2.2.3; VIM(2), 3.9]

> *EDITORIAL NOTE: [GUM] still utilizes this definition according to [VIM(2)] which was withdrawn. Here, the measurement uncertainty is assigned to the measurement result whereas it is assigned to the measurand according to its revised definition [VIM, 2.26].*

measurement uncertainty *(Ger. Messunsicherheit)*
Parameter obtained from **measurements** and which – together with the **result of measurement** – characterizes the range of values within which the **true value** of a **measurand** is estimated to lie [DIN 1319-1, 3.6]

> *NOTE 2: The measurement uncertainty has to be distinguished clearly from the **measurement error**. A measurement error merely is the difference between a value which is assigned to a measurand, e.g. a measured value or a measurement result, and the true value of the measurand. The measurement error may be zero without being known. This lack of knowledge is expressed in a measurement uncertainty which is greater than zero.*

[DIN 1319-4, 3.5]; note 2 loosely translated from German, official English translation unavailable

measurement unit *(Ger. Maßeinheit)*
real scalar **quantity**, defined and adopted by convention, with which any other quantity of the same **kind** can be compared to express the ratio of the two quantities as a number

> *NOTE 1: Measurement units are designated by conventionally assigned names and symbols.*

[VIM, 1.9]

measuring equipment *(Ger. Messmittel)*
measuring instrument, software, **measurement standard**, reference material or auxiliary apparatus or combination thereof necessary to realize a **measurement process** [ISO 9000, 3.11.16]

measuring instrument *(Ger. Messgerät)*
device used for making **measurements**, alone or in conjunction with one or more supplementary devices

> *NOTE 1: A measuring instrument that can be used alone is a **measuring system**.*
>
> *NOTE 2: A measuring instrument may be an **indicating measuring instrument** or a **material measure**.*

[VIM, 3.1]

measuring object *(Ger. Messobjekt)*
The object being measured in order to determine the value of the **measurand** [DIN 1319-1, 1.2]

measuring system *(Ger. Messsystem)*
set of one or more **measuring instruments** and often other devices, including any reagent and supply, assembled and adapted to give information used to generate **measured quantity values** within specified intervals for **quantities** of specified **kinds**

NOTE: A measuring system may consist of only one measuring instrument.

[VIM, 3.2]

metrological compatibility *(Ger. metrologische Verträglichkeit)*
property of a set of **measurement results** for a specified **measurand**, such that the absolute value of the difference of any pair of **measured quantity values** from two different measurement results is smaller than some chosen multiple of the **standard measurement uncertainty** of that difference [VIM, 2.47]

nominal property *(Ger. Nominalmerkmal)*
property of a phenomenon, body, or substance, where the property has no magnitude [VIM, 1.30]

nominal value *(Ger. Nominalwert):* see **target value**

observed value *(Ger. Beobachteter Wert)*
obtained value of a property associated with one member of a **sample** [ISO 3534-1, 1.4]

population *(Ger. Grundgesamtheit)*
totality of **items** under consideration [ISO 3534-2, 1.2.1]

quantity *(Ger. Größe)*
property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference [VIM, 1.1]

quantity value *(Ger. Größenwert)*
number and reference together expressing magnitude of a **quantity** [VIM, 1.19]

random (measurement) error *(Ger. zufällige Messabweichung)*
component of **measurement error** that in replicate **measurements** varies in an unpredictable manner

*NOTE 1: A **reference quantity value** for a random measurement error is the average that would ensue from an infinite number of replicate measurements of the same **measurand**.*

NOTE 2: Random measurement errors of a set of replicate measurements form a distribution that can be summarized by its expectation, which is generally assumed to be zero, and its variance.

*NOTE 3: Random measurement error equals measurement error minus **systematic measurement error**.*

[VIM, 2.19]

random sample *(Ger. Zufallsstichprobe)*
sample which has been selected by a method of random selection [ISO 3534-1, 1.6]

<u>**reference (quantity) value**</u> *(Ger. Referenzwert)*
quantity value used as a basis for comparison with values of **quantities** of the same **kind**

*NOTE 1: A reference quantity value can be a **true quantity value** of a **measurand**, in which case it is unknown, or a **conventional quantity value**, in which case it is known.*

*NOTE 2: A reference quantity value with associated **measurement uncertainty** is usually provided with reference to*
a) a material, e.g. a certified reference material,
b) a device, e.g. a stabilized laser,
c) a reference measurement procedure,
*d) a comparison of **measurement standards**.*

[VIM, 5.18]

<u>**relative standard (measurement) uncertainty**</u> *(Ger. relative Standard(mess)unsicherheit)*
standard measurement uncertainty divided by the absolute value of the **measured quantity value** [VIM, 2.32]

<u>**repeatability condition** (of measurement)</u> *(Ger. Wiederholbedingung)*
condition of **measurement**, out of a set of conditions that includes the same **measurement procedure**, same operators, same **measuring system**, same operating conditions and same location, and replicate measurements on the same or similar objects over a short period of time [VIM, 2.20]

<u>**reproducibility condition**</u> *(Ger. Erweiterte Vergleichbedingung)*
condition of measurement, out of a set of conditions that includes different locations, operators, **measuring systems**, and replicate measurements on the same or similar objects [VIM, 2.24]

<u>**requirement**</u> *(Ger. Anforderung)*
need or expectation that is stated, generally implied or obligatory [ISO 9000, 3.6.4]

<u>**resolution**</u> *(Ger. Auflösung)*
smallest change in a **quantity** being measured that causes a perceptible change in the corresponding **indication** [VIM, 4.14]

<u>**sample**</u> *(Ger. Stichprobe)*
subset of a **population** made up of one or more **sampling units** [ISO 3534-2, 1.2.17]

<u>**sampling unit**</u> *(Ger. Auswahleinheit)*
one of the individual parts into which a **population** is divided

NOTE 1: A sampling unit can contain one or more items, for example a box of matches, but one test result will obtained for it.

[ISO 3534-2, 1.2.14]

<u>**specification**</u> *(Ger. Spezifikation)*
document stating **requirements**

Note 1: A specification can be related to activities (e.g. procedure document, process specification and test specification), or products (e.g. product specification, performance specification and drawing).

[ISO 9000, 3.8.7]

EDITORIAL NOTE: In everyday language "to specify" usually means determining (e.g. by measurements), stating (e.g. based on evluation results) and documenting requirements.

<u>**specification interval**</u> *(Ger. Spezifikationsintervall)*
interval between upper and lower **specification limits** [ISO 22514-1, 3.1.14]

<u>**specification limit**</u> *(Ger. Grenzwert)*
limiting value stated for a **characteristic** [ISO 3534-2, 3.1.3]

stability (of a measuring instrument) *(Ger. Messbeständigkeit)*
property of a **measuring instrument**, whereby its metrological properties remain constant in time [VIM, 4.19]

standard (measurement) uncertainty *(Ger. Standard(mess)unsicherheit)*
measurement uncertainty expressed as a standard deviation [VIM, 2.30]

statistic *(Ger. Kenngröße)*
completely specified function of random variables [ISO 3534-1, 1.8]

systematic (measurement) error *(Ger. systematische Messabweichung)*
component of **measurement error** that in replicate **measurements** remains constant or varies in a predictable manner

NOTE 1: A ***reference quantity value*** *for a systematic measurement error is a* ***true quantity value****, or a* ***measured quantity value*** *of a* ***measurement standard*** *of negligible* ***measurement uncertainty****, or a* ***conventional quantity value****.*

NOTE 3: Systematic measurement error equals ***measurement error*** *minus* ***random measurement error****.*

[VIM, 2.17]

target value *(Ger. Sollwert)*
preferred or reference value of a **characteristic** stated in a **specification** [ISO 3534-2, 3.1.2]

(specified) **tolerance** *(Ger. (festgelegte) Toleranz)*
difference between the **upper specification limits** and **lower specification limits** [ISO 3534-2, 3.1.6]

tolerance interval *(Ger. Toleranzintervall)*
see **specification interval**

tolerance zone *(Ger. Toleranzzone)*
see **specification interval**

true (quantity) value *(Ger. wahrer Wert einer Größe)*
quantity value consistent with the definition of a **quantity** [VIM, 2.11]

true value *(Ger. wahrer Wert)*
value which characterizes a **quantity** or quantitative **characteristic** perfectly defined in the conditions which exist when that quantity or quantitative characteristic is considered

NOTE 1: The true value of a quantity or a quantitative characteristic is a theoretical concept and, in general, cannot be known exactly.

[ISO 3534-2, 3.2.5]

Type A evaluation *(Ger. Ermittlungsmethode A)*
evaluation of a component of **measurement uncertainty** by a statistical analysis of **measured quantity values** obtained under defined measurement conditions

NOTE 1: For various types of measurement conditions, see ***repeatability condition of measurement, intermediate precision condition of measurement****, and* ***reproducibility condition of measurement****.*

[VIM, 2.28]

Type B evaluation *(Ger. Ermittlungsmethode B)*
evaluation of a component of **measurement uncertainty** determined by means other than a **Type A evaluation of measurement uncertainty** [VIM, 2.29]

uncertainty budget *(Ger. Messunsicherheitsbilanz)*
statement of a **measurement uncertainty**, of the components of that measurement uncertainty, and of their calculation and combination

> *NOTE: An uncertainty budget should include the **measurement model**, **estimates**, and measurement uncertainties associated with the **quantities** in the measurement model, covariances, type of applied probability density functions, **degrees of freedom**, type of evaluation of measurement uncertainty, and any **coverage factor**.*

[VIM, 2.33]

upper specification limit *(Ger. Höchstwert)*
specification limit that defines the upper limiting value [ISO 3534-2, 3.1.4]

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